

39 Refrigeration, heat pump and air conditioning

39.1 Introduction

A *heat pump* is a machine that moves heat from one region (the *source*) at a lower temperature (T_L) to another region (the *heat sink*) at a higher temperature (T_H) using mechanical work or a high-temperature heat source. Thus a heat pump may be thought as a “heater” if the aim is to warm up the heat sink, as when warming the inside of a home on a cold day, or a “refrigerator” if the aim is to cool the heat source, as in a freezer. In either case, the operating principles are identical: heat is moved from a cold place into a warm place.

Thus, refrigeration should be considered as a special aspect of heat transfer; it requires the production and utilization of below-atmospheric temperature sources by a number of processes. A substance is cooled when the heat is transferred, via a temperature drop, to solid, liquid, or gaseous media which are naturally or artificially colder^{39.1}. The most commonly used refrigeration cycle is the *vapor-compression cycle* where the refrigerant is cooled through the pressure drop due to a throttling process (*Paragraph 31.3*) taking place through an expansion valve.

Air conditioning is another aspect of heat transfer. It involves, as a working fluid, the atmospheric air, i.e. a mixture of air and water vapor, to influence the human comfort by changing temperature and relative humidity of the air. The air conditioning principles are the same as those used to design the evaporative towers, a very efficient means to condense vapor.

39.2 Refrigeration and refrigerants

Refrigeration is the process of extracting the heat q_L from the low temperature region and releasing the heat q_H to the high temperature region. Since the heat flows spontaneously from high- to low-temperature regions according to the second law of thermodynamics (*Paragraph 28.8*), refrigeration absorbs power to force the heat transfer to occur. The energy source is a pump or compressor that supplies the work needed to compress the refrigerant. The resulting cycle is very much like the Rankine cycle run in reverse.

In the ideal cycle for vapor compression refrigeration (*Figure 39.1*):

1 \Rightarrow 2 saturated vapor at low pressure enters the *compressor* and undergoes a reversible adiabatic compression (*isentropic compression*);

39.1 - In general, the refrigeration machines can be classified as:

1. vapor-compression refrigeration cycle;
2. air-standard refrigeration cycle;
3. absorption refrigeration cycle;
4. thermoelectric refrigeration effect.

- 2 \Rightarrow 3 the heat q_H is rejected to the high temperature region in a *condensation* process at *constant pressure* through a heat exchanger, the *condenser*, and the working fluid leaves the condenser as saturated liquid;
- 3 \Rightarrow 4 an adiabatic *throttling* process through the *expansion valve* where the fluid passes from the high to the low pressure, and simultaneously reducing its temperature while maintaining the initial enthalpy (*isenthalpic expansion*);
- 4 \Rightarrow 1 the heat q_L is received by the low temperature region and the working fluid is *evaporated* at a *constant pressure* process through a heat exchanger, the *evaporator*, to complete the cycle.

The refrigeration cycle looks quite similar both on a process scheme and on a T - s diagram (Figure 39.1) to the Rankine cycle (Figure 35.1): it is essentially the same cycle in reverse except that an expansion valve replaces the pump and the work has to be supplied to the refrigeration cycle via a compressor while in the Rankine cycle work is produced by a turbine. By using the valve, the cycle can never be reversible, since the throttling process is inherently irreversible. Thus the major difference is that the reversed cycle is irreversible, whereas the Rankine cycle with its pumping process is reversible. Furthermore, the ideal refrigeration cycle 1-2-3-4-1 deviates from the Carnot cycle 1'-2'-3-4'-1': at low temperature (4 \Rightarrow 1 instead of 4' \Rightarrow 1') and at high temperature (2 \Rightarrow 3 instead of 2' \Rightarrow 3)^{39.2}.

The *coefficient of performance* COP_F for the vapor-compression refrigeration cycle is given by:

$$COP_F = \frac{q_L}{w} \quad \mathbf{39-1}$$

$$q_L = h_1 - h_4 \quad \mathbf{39-2}$$

$$w = w_i = h_2 - h_1 \quad \mathbf{39-3}$$

where q_L , *refrigeration effect*, is the heat extracted from low temperature region by the evaporator and defines the amount of cooling produced by a system;

w is the work entering the cycle and corresponds to the internal work w_i absorbed by the compressor.

From the first law of thermodynamics, the absorbed work w is connected to the heat q_H released by the condenser and the heat q_L extracted by the evaporator:

$$w = q_H - q_L \Rightarrow q_H = q_L + w \quad \mathbf{39-4}$$

$$q_H = h_2 - h_3 \quad \mathbf{39-5}$$

with q_H given by the enthalpy difference to de-superheat and subsequently condense the vapor in the process 2 \Rightarrow 3. Finally it should be reminded that power P [$W = J/s$], both as mechanical power \dot{W} or heat power \dot{Q} , is obtained by multiplying the mass flow rate of the working fluid \dot{m} [kg/s] for the specific work w [J/kg] or the specific heat q [J/kg]. Thus, the relationship $\dot{Q} = \dot{m}q$ is used to obtain the working fluid flow rate needed to extract the heat power \dot{Q}_L from the low temperature region with refrigeration machine or to supply the heat

39.2 - The deviation between the ideal refrigeration cycle 1-2-3-4-1 and the Carnot cycle 1'-2'-3-4'-1' is an expedient to have a compressor handling only a vapor than a liquid-vapor mixture as would be required in Carnot cycle process 1' \Rightarrow 2'. In fact, it is practically impossible to compress at a reasonable rate the liquid-vapor mixture of state 1' and maintain equilibrium between the two liquid and vapor phases because of heat and mass transfer rate typical of phase change. It is also much simpler to have the expansion process taking place irreversibly via throttling than to have an expansion device that receives saturated liquid and discharges a liquid-vapor mixture, as required in process 3 \Rightarrow 4'.

power \dot{Q}_H to the high temperature region with heat pump, while the product $\dot{m}w$ allows to obtain the power P absorbed by the compressor.

$$\dot{Q} = \dot{m}q \Rightarrow \dot{m} = \frac{\dot{Q}}{q} \qquad P = \dot{m}w \qquad \mathbf{39-6}$$

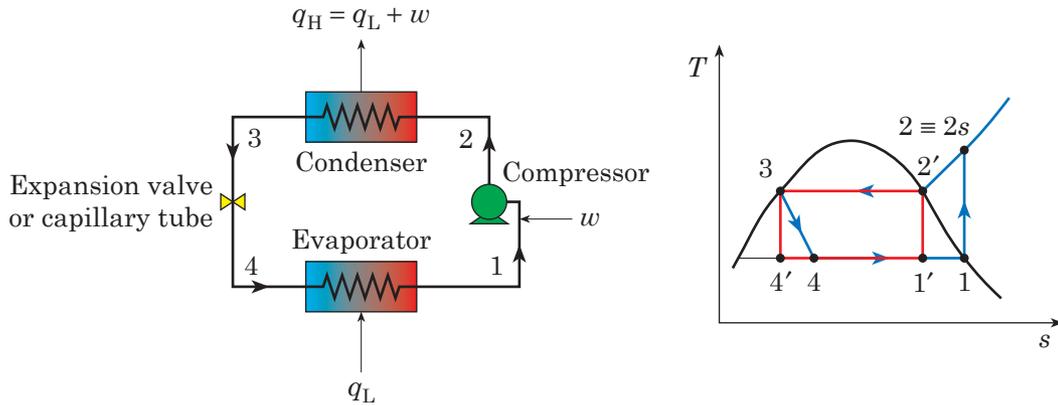


Fig. 39.1 - The ideal vapor-compression refrigeration cycle.

The *reversed Carnot cycle* operating between a refrigerated region at T_L and the atmosphere at T_H is usually represented in a pressure-enthalpy diagram that, much better than T - s or Mollier h - s diagrams, describes (*Figure 39.1*) the constant pressure processes $2 \Rightarrow 3$ and $4 \Rightarrow 1$ (horizontal lines), and the constant enthalpy process $3 \Rightarrow 4$ (vertical line). *Figure 39.2* shows the skeleton of a p - h diagram with:

- *vapor dome* including the two-phase liquid-vapor mixture. Its left-hand part (*saturated liquid line*) separates the liquid phase from the liquid-vapor phase. Similarly, the right-hand of the line (*saturated vapor line*) separates the liquid-vapor phase from the vapor phase. At the highest point of the vapor dome the critical point is shown;
- dashed and dotted black lines of constant quality x , red lines of constant temperature T , orange lines of constant entropy s , green lines of constant specific volume v , and lines at constant pressure that, within the vapor dome, are superimposed on the constant temperature lines;
- purple lines of the refrigeration cycle $1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 4 \Rightarrow 1$.

The use of p - h diagram of refrigerant R134a^{39.3}, see *Paragraph 29.5* of the second volume, and *Table A.4*, with its thermodynamic properties, will be illustrated in the Examples.

39.3 - It was already pointed out that in thermodynamics only the difference of entropy or enthalpy are considered; for instance, work (31-5) and heat (31-6) can be represented by the difference of enthalpy. Being only the property change of interest, it is the same to refer the entropy or enthalpy origin to a state or another one. Then, on p - h diagram, described in *Paragraph 29.5* of the second volume, as well in *Table A.4*, enthalpy and entropy of saturated liquid were fixed as $h_f = 200.00$ kJ/kg and $s_f = 1.0000$ kJ/(kg·K) respectively at a temperature of 0 °C.

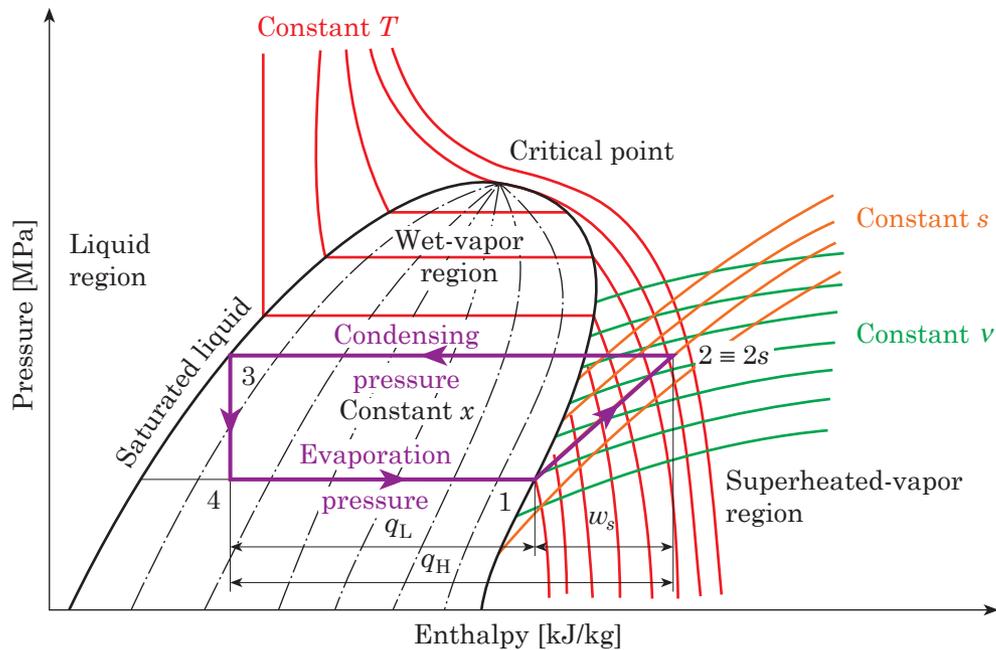


Fig. 39.2 - Ideal vapor-compression cycle on p - h diagram of a typical refrigerant.

Besides not to be toxic, irritant and flammable, and no damaging to the environment, a *refrigerant fluid* should be characterized by:

- a high critical temperature much higher than the condensation temperature of the cycle;
- a low fusion temperature to avoid solidification in normal operating condition;
- a very high heat of vaporization to generate a high refrigeration effect q_L ;
- a stable chemical composition.

The halogenated hydrocarbons (Freon), mostly used in the last century, are divided in three groups:

- CFC (chlorofluorocarbons), with chlorine fully substituting hydrogen in the molecule;
- HCFC (hydrochlorofluorocarbons), with chlorine, fluorine and hydrogen atoms;
- HFC (hydrofluorocarbons), without chlorine.

Because CFCs contribute to ozone depletion potential (ODP) in the upper atmosphere, the manufacture of such compounds has been phased out under the Montreal Protocol, and they are being replaced with other products such as HFCs, hydrocarbons, and CO_2 . A today common refrigerant is the R134a, that, besides fluorine (F) and carbon (C), contains hydrogen (H) atoms; its global warming potential (GWP) is not very high (*Table 39.1*), it is not toxic and flammable and does not etch copper and its alloys. The lower are ODP and GWP values the better is environment protection.

The ammonia is very interesting: its refrigeration effect q_L is extremely high as shown in *Table 39.2*; however, ammonia is toxic and flammable. The carbon dioxide (CO_2) is an excellent refrigerant because of its low boiling point; in fact, even if the ambient temperature is below $0^\circ C$, it can boil and evaporate; furthermore it is non-flammable, non-ozone

Table 39.1

Characteristics of refrigerants

Refrigerant	Formula	Boiling point at 101.32 kPa [°C]	Melting point [°C]	Critical temperature [°C]	Critical pressure (absolute) [MPa]	ODP (*)	GWP (**)
R11, Trichlorofluoromethane	CFCl ₃	+23.7	-111	+198.0	4.37	1.0	4750
R134a, Tetrafluoroethane	C ₂ H ₂ F ₄	-26.3	-103.3	+101.1	4.06	0	1430
R717, Ammonia	NH ₃	-33.35	-77.9	+132.4	11.3	0	0
R744, Carbon dioxide	CO ₂	-78.5	-56.9	+31.1	73.8	0	1

(*) ODP (Ozone depletion potential): the potential of a single refrigerant molecule to destroy the ozone layer (Paragraph 12.2.3). Reference is R11 with ODP = 1.0.

(**) GWP (Global warming potential): how much refrigerant affects, in a time frame of 100 years, the global warming. Reference is carbon dioxide (CO₂) with GWP = 1.

Table 39.2

Performance of refrigerants

Ideal vapor-compression refrigeration cycle having as evaporation temperature $T_L = -15$ °C and as condensation temperature $T_H = +30$ °C.

Refrigerant	Evaporation pressure $p_1 = p_4$ [MPa]	Condensation pressure $p_2 = p_3$ [MPa]	Specific volume of saturated vapor v_1 [dm ³ /kg]	Refrigeration effect (*)		Ideal COP _F (*) [-]
	q_L [kJ/kg]	q_1/v_1 [kJ/m ³]				
Ammonia	0.236	1.166	509	1102.6	2166.2	4.75
R11	0.021	0.089	772	157.2	203.6	5.14
R134a	0.1642	0.7698	120	147.0	1223	4.7

(*) Refrigeration effect and ideal coefficient of performance of R134a are calculated, according to the ideal vapor-compression cycle shown in Figure 39.3 (see Example 39.1):

$$\text{Table A.4.1 } (T_L = -15 \text{ °C}): h_1 = h_g = 388.56 \frac{\text{kJ}}{\text{kg}}; \quad s_1 = s_g = 1.7330 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}; \quad v_1 = v_g = 120.15 \frac{\text{dm}^3}{\text{kg}}$$

$$\text{Table A.4.1 } (T_H = +30 \text{ °C}): h_4 = h_3 = h_f = 241.61 \text{ kJ/kg}$$

$$\text{R134a diagram: } s_{2s} = s_1 = 1.7330 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}, \quad T = 30 \text{ °C} \Rightarrow h_2 = h_{2s} = 419.8 \text{ kJ/kg}$$

$$\mathbf{39-2: } q_L = h_1 - h_4 = 388.56 \text{ kJ/kg} - 241.61 \text{ kJ/kg} = 146.95 \text{ kJ/kg} \approx 147 \text{ kJ/kg}$$

$$\frac{q_L}{v_1} = \frac{147 \text{ kJ/kg}}{0.12015 \text{ m}^3/\text{kg}} = 1223 \frac{\text{kJ}}{\text{m}^3}$$

$$\mathbf{39-3, Figure 39.3: } w = w_1 = w_s = h_{2s} - h_1 = 419.8 \text{ kJ/kg} - 388.56 \text{ kJ/kg} = 31.2 \text{ kJ/kg}$$

$$\mathbf{39-1: } COP_F = \frac{q_L}{w} = \frac{h_1 - h_4}{h_{2s} - h_1} = \frac{147 \text{ kJ/kg}}{31.2 \text{ kJ/kg}} = 4.7$$

depleting, and has a global warming potential of 1. CO₂ can be used as a working fluid in climate control systems for cars, residential air conditioning, hot water pumps, and commercial refrigeration.

39.3 Ideal refrigeration cycle

Figure 39.3 illustrates on pressure-enthalpy diagram the ideal vapor-compression refrigeration cycle shown in Figure 39.1.

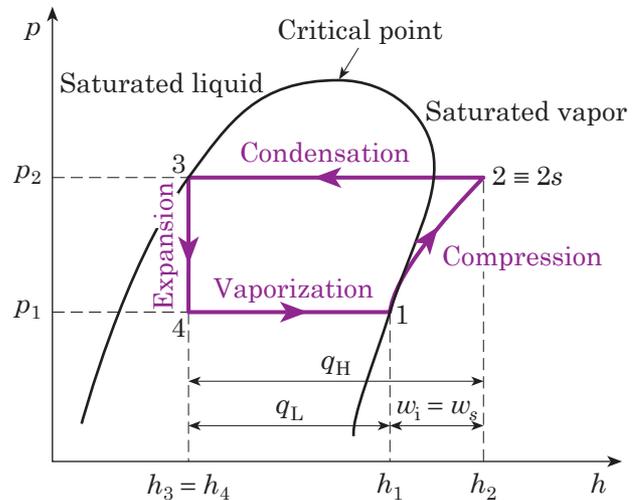


Fig. 39.3 - Ideal vapor-compression refrigeration cycle.

Read on the abscissae:

- the refrigeration effect $q_L = h_1 - h_4$ (39-2);
- the compression work $w_i = w_s = h_2 - h_1$ (39-3), that is done along the constant entropy line $1 \Rightarrow 2s$;
- the heat rejected from the condenser $q_H = h_2 - h_3$ (39-4), at first in desuperheating phase at constant pressure, then at constant pressure and temperature during condensation within the vapor dome;
- the point 4, end of throttling in the isenthalpic process $3 \Rightarrow 4$ from the saturated liquid point 3 down to the horizontal line $4 \Rightarrow 1$ of constant pressure evaporation.

Once known the mass flow rate \dot{m} of the working fluid and read the specific volume ν_1 [m³/kg] on p - h diagram, the volume flow rate \dot{V}_1 needed to design the compressor is:

$$\dot{V}_1 = \dot{m} \nu_1 \quad \mathbf{39-7}$$

The evaporation and condensation temperature are necessarily different from those of the space where heat is extracted and released respectively. In fact evaporator and condenser are heat exchangers; therefore to allow the heat-transfer the working fluid during evaporation should be at a lower temperature than that of the space (the *source*) from which the heat is extracted, while during condensation the fluid has to be at higher temperature than that of the space where the heat is released (the *heat sink*). In general the temperature difference is at least about 10 °C; thus, if, for instance, the temperature of food store of

Example 39.2 has to be maintained at $-10\text{ }^{\circ}\text{C}$ with the surroundings at $+20\text{ }^{\circ}\text{C}$, the evaporation temperature of the fluid in the cycle must be less than or equal to $-20\text{ }^{\circ}\text{C}$, while the condensation temperature must be higher than or equal to $+30\text{ }^{\circ}\text{C}$. This means that temperature difference between the source (the food store) and the heat sink (the surroundings) is $30\text{ }^{\circ}\text{C}$, while the difference between the evaporation and condensation temperatures of the fluid is equal at least to $50\text{ }^{\circ}\text{C}$.

In general the temperature difference is maintained as low as possible because of energy saving. However, this criterion can be modified by other considerations of overall dimensions and working conditions of evaporator and condenser. For instance, in the window air conditioner of *Example 39.1*, the room from which the evaporator extracts heat can be at $25\text{ }^{\circ}\text{C}$ and the fluid can evaporate at $0\text{ }^{\circ}\text{C}$, while the outdoor air can be at $35\text{ }^{\circ}\text{C}$ and the fluid can condense at $50\text{ }^{\circ}\text{C}$; then a $10\text{ }^{\circ}\text{C}$ temperature difference between room and outdoor air is amplified until $50\text{ }^{\circ}\text{C}$ in the cycle.

Example 39.1 Window air conditioner

In a 5 kW window air conditioner, the R134a working fluid describes the cycle of *Figure 39.4* between an evaporation temperature equal to $0\text{ }^{\circ}\text{C}$ and a condensation temperature equal to $50\text{ }^{\circ}\text{C}$. Determine:

- the coefficient of performance COP_F ;
- the mass flow rate \dot{m} of refrigerant;
- the compressor volume flow rate \dot{V}_1 ;
- the ideal power P_{id} absorbed by the compressor.

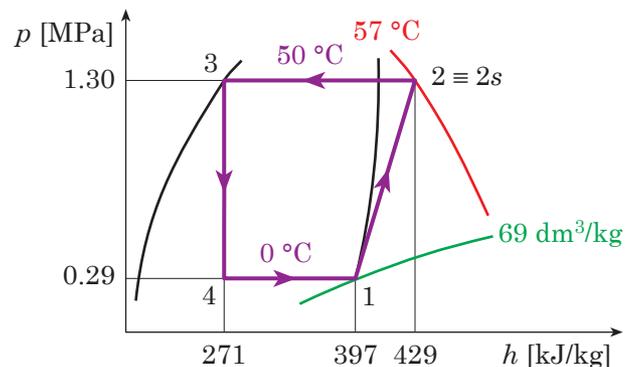


Fig. 39.4 - Cycle of *Example 39.1*.

SOLUTION

- The ideal coefficient of performance of R134a is calculated according to the ideal vapor-compression cycle shown in *Figure 39.3*:

STATE 1 – Saturated vapor, $T_L = 0\text{ }^{\circ}\text{C}$, *Table A.4.1*:

$$p_1 = 0.2929\text{ MPa}; \quad h_1 = h_g = 397.56 \frac{\text{kJ}}{\text{kg}}; \quad v_1 = v_g = 69.01 \frac{\text{dm}^3}{\text{kg}}; \quad s_1 = s_g = 1.7233 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

STATE 2 – Intersection of isentropic line [$s_{2s} = s_1 = 1.7233 \text{ kJ}/(\text{kg}\cdot\text{K})$] with isobaric line $p_2 = 1.3171 \text{ MPa}$, that is (*Table A.4.1*) the saturation pressure corresponding to condensation temperature $T_H = +50 \text{ }^\circ\text{C}$:

$$h_2 = h_{2s} = 429 \text{ kJ/kg}; \quad T_2 = 57 \text{ }^\circ\text{C} \text{ (slightly superheated vapor)}$$

STATE 3 – Saturated liquid, $T_H = +50 \text{ }^\circ\text{C}$, *Table A.4.1*: $h_3 = h_f = 271.02 \text{ kJ/kg}$

STATE 4 – Being the throttling process isenthalpic, h_4 is equal to h_3 : $h_4 = h_3 = 271.02 \text{ kJ/kg}$

$$\mathbf{39-2:} \quad q_L = h_1 - h_4 = 397.56 \text{ kJ/kg} - 271.02 \text{ kJ/kg} = 126.5 \text{ kJ/kg}$$

$$\mathbf{39-3:} \quad w = w_i = w_s = h_2 - h_1 = 429 \text{ kJ/kg} - 397.56 \text{ kJ/kg} = 31.5 \text{ kJ/kg}$$

$$\mathbf{39-1:} \quad COP_F = \frac{q_L}{w} = \frac{126.5 \text{ kJ/kg}}{31.5 \text{ kJ/kg}} = 4.0 \quad \blacktriangleleft$$

b) The heat power to be extracted is known: $\dot{Q}_L = 5 \text{ kW}$. The refrigerant mass flow rate is:

$$\mathbf{39-6:} \quad \dot{Q} = \dot{m}q \Rightarrow \dot{m} = \frac{\dot{Q}}{q} = \frac{5 \text{ kW}}{126.5 \text{ kJ/kg}} = 0.0395 \frac{\text{kg}}{\text{s}} \quad \blacktriangleleft$$

c) The refrigerant volume flow rate required to design the compressor is:

$$\mathbf{39-7:} \quad \dot{V}_1 = \dot{m}v_1 = 0.0395 \frac{\text{kg}}{\text{s}} \times 0.06901 \frac{\text{m}^3}{\text{kg}} = 0.0027 \frac{\text{m}^3}{\text{s}} \quad \blacktriangleleft$$

d) The ideal power absorbed by compressor is:

$$\mathbf{39-6:} \quad P_{\text{id}} = \dot{m}w_s = 0.0395 \frac{\text{kg}}{\text{s}} \times 31.5 \frac{\text{kJ}}{\text{kg}} = 1.2 \text{ kW} \quad \blacktriangleleft$$

COMMENT The value of the coefficient of performance of the reversed Carnot cycle operating between the same absolute temperatures as those of the cycle, i.e. evaporation $T_L = 0 \text{ }^\circ\text{C} = 273.15 \text{ K}$ and condensation $T_H = 50 \text{ }^\circ\text{C} = 323.15 \text{ K}$, represents the theoretical limit of the process. By comparing the value $COP_F = 4.0$, obtained in the Example, with the coefficient of performance of the reversed Carnot cycle, it is found:

$$\mathbf{30-4:} \quad COP_{F_Carnot} = \frac{T_L}{T_H - T_L} = \frac{273.15 \text{ K}}{323.15 \text{ K} - 273.15 \text{ K}} = 5.5$$

/// 39.4 Actual refrigeration cycle

The actual refrigeration cycle deviates from the ideal cycle, shown in *Figure 39.3*, primarily because of pressure drops associated with fluid flow in the piping and heat transfer to or from the surroundings. The vapor entering the compressor (*Figure 39.5-a*) is slightly superheated (state 1 instead of 1' of saturated vapor) to avoid that fluid sucked from com-

pressor contains some liquid that, being incompressible, can cause problem at the end of the compression stroke in a reciprocating compressor. *Figure 39.5-a* shows as well a slight subcooling: the working fluid releases heat in the condenser not only until the full condensation ($2 \Rightarrow 3'$), but it further cools ($3' \Rightarrow 3$) at the same pressure. The model cycle, shown in *Figure 39.5-b*, is usually utilized: besides the slight superheating of vapor (state 1), the slight deviation of the actual compression ($1 \Rightarrow 2$) in respect of the ideal isentropic compression ($1 \Rightarrow 2s$)^{39.4} is then taken into consideration.

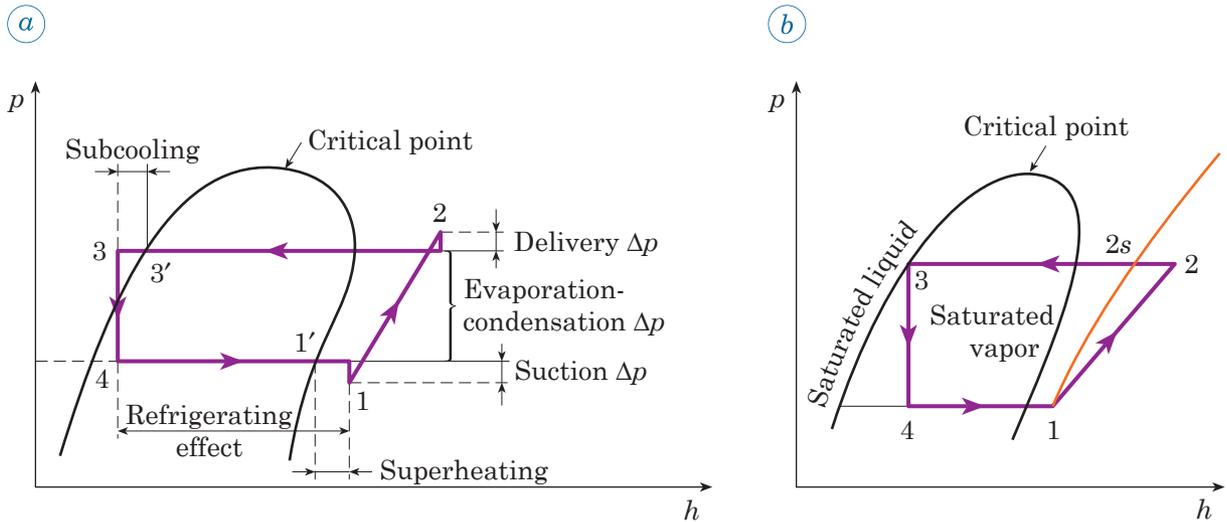


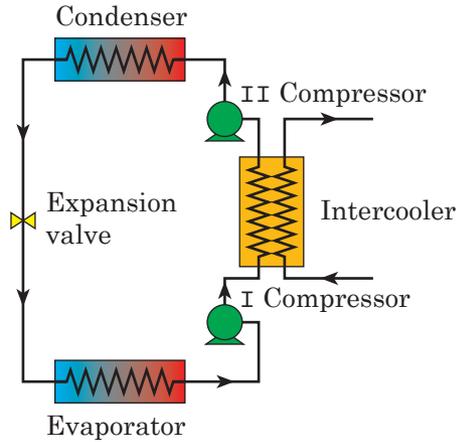
Fig. 39.5 - The actual vapor-compression refrigeration cycle in the $p-h$ diagram:

- delivery and suction pressure drop;
- isentropic compression line ($1 \Rightarrow 2s$) and the actual one ($1 \Rightarrow 2$).

Often, there is a need to achieve a high temperature difference between evaporation and condensation; this difference has to be associated to a high final-to-inlet pressure ratio of the compressor. The problem is solved by the use of two-stage compression with intercooling. With respect to one-stage compression with $2''$ as end-point, the two-stage compression allows to reach lower temperatures at the end of compression ($T_2 < T_{2''}$) with a lower compression work [$w = (h_{1'} - h_1) + (h_2 - h_{2'})$ instead of $w = h_{2''} - h_1$] because of intermediate cooling (*Figure 39.6-a*) able to reduce temperature from $T_{1'}$ to $T_{2'}$ (*Figure 39.6-b*).

^{39.4} - During the compression process there are irreversibilities with heat transfer occurring either to or from the surroundings, depending on the temperature of the refrigerant and the surroundings. Therefore, the entropy might increase or decrease during the process: irreversibility and heat transfer to the refrigerant cause an entropy increase, while heat transfer from the refrigerant cause an entropy decrease. Thus, in a first approximation calculation, the compression process is still assumed to be an isentropic one.

a)



b)

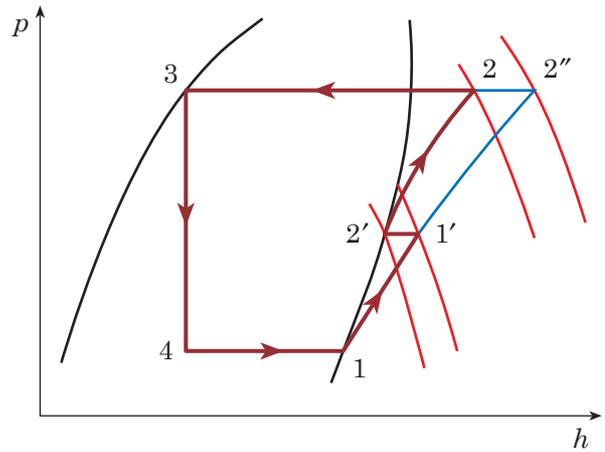


Fig. 39.6 - Two-stage compression with intercooling:

a) the system;

b) the process in the p - h diagram.

Example 39.2 Food cold store

A food store has to be maintained at $-10\text{ }^\circ\text{C}$ with the outside air at $+20\text{ }^\circ\text{C}$. The heat power to be extracted is 100 kW . Taking into account the temperature difference between the working fluid and the two areas (the food store and the outside air), fix a condensation temperature equal to $+30\text{ }^\circ\text{C}$ and an evaporation temperature of the refrigerant (R134a) equal to $-20\text{ }^\circ\text{C}$. By following the normal practice, fluid enters the compressor as $5\text{ }^\circ\text{C}$ superheated vapor, i.e. with $5\text{ }^\circ\text{C}$ above the saturation temperature corresponding to the intake pressure in the compressor. Determine:

- the main properties of the cycle points (the internal efficiency of the reciprocating compressor is $\eta_c = 0.92$);
- the heat q_L extracted through the evaporator and q_H withdrawn from the condenser, and the coefficient of performance COP_F ;
- the mass flow rate \dot{m} of refrigerant, the total displacement V_t of a V 6-cyl compressor [rotational speed $n = 1200\text{ rpm}$ (revolutions per minute), volumetric efficiency $\eta_v = 0.75$] and the power P_a absorbed by a compressor having a mechanical efficiency $\eta_m = 0.95$.

State	$T\text{ [}^\circ\text{C]}$	$p\text{ [MPa]}$	$h\text{ [kJ/kg]}$	$s\text{ [kJ/kg}\cdot\text{K]}$	$\nu\text{ [dm}^3\text{/kg]}$	Fluid [R134a]
1	-15	0.133	390	1.755	150	Superheated vapor
2s	+43	0.7698	427	1.755	28	Superheated vapor
2	+47	0.7698	430.2	1.762	29	Superheated vapor
3	+30	0.7698	241.61	1.1431	0.842	Saturated liquid
4	-20	0.133	241.61	1.17	55	Liquid-vapor ($x = 0.32$)

Red values are obtained from p - h diagram, while black values are read on *Table A.4*.

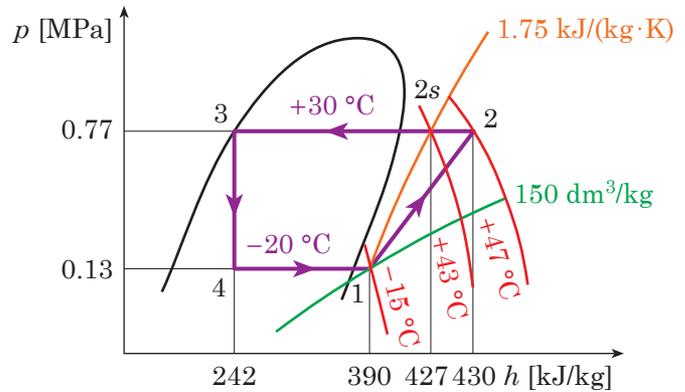


Fig. 39.7 - Cycle of *Example 39.2*.

SOLUTION

a) *Figure 39.7* shows the properties of each point of the cycle.

STATE 1 – Read on *Table A.4.1* the saturation pressure $p_1 = 0.133$ MPa for the evaporation temperature of -20 °C. The refrigerant is slightly superheated at the temperature $T_1 = -15$ °C. Point 1 is the intersection of -15 °C isotherm (obtained by interpolation of the two red curves at -20 and -10 °C) with $p_1 = 0.133$ MPa horizontal line on p - h diagram. Read: enthalpy $h_1 = 390$ kJ/kg, entropy $s_1 = 1.755$ kJ/(kg·K) and specific volume $v_1 = 150$ dm³/kg = 0.15 m³/kg.

STATE 2s – Being an isentropic compression, $s_{2s} = s_1 = 1.755$ kJ/(kg·K). Vapor pressure $p_{2s} = p_2$ equals $p_3 = 0.7698$ MPa, that is the saturation pressure corresponding to the condensation temperature $T_3 = +30$ °C. Then moving on p - h diagram along the isentropic orange line from 1 until the horizontal line with pressure $p_3 = p_{2s} = 0.7698$ MPa, point 2s is located. Read: $T_{2s} = +43$ °C, enthalpy $h_{2s} = 427$ kJ/kg and specific volume $v_{2s} = 28$ dm³/kg = 0.028 m³/kg.

STATE 2 – At first the ideal compression work w_s is calculated. Taking into account the internal compression efficiency $\eta_c = 0.92$, the internal work w_i and then the actual enthalpy h_2 are obtained.

$$\mathbf{39-3:} \quad w_s = h_{2s} - h_1 = 427 \text{ kJ/kg} - 390 \text{ kJ/kg} = 37 \text{ kJ/kg}$$

$$\mathbf{31-8:} \quad \eta_c = \frac{w_s}{w_i} \Rightarrow w_i = \frac{w_s}{\eta_c} = \frac{37 \text{ kJ/kg}}{0.92} = 40.2 \frac{\text{kJ}}{\text{kg}}$$

$$\mathbf{39-3:} \quad w_i = h_2 - h_1 \Rightarrow h_2 = w_i + h_1 = 40.2 \text{ kJ/kg} + 390 \text{ kJ/kg} = 430.2 \text{ kJ/kg}$$

The point 2, that represents the fluid condition leaving the compressor, is found as intersection, on p - h diagram, between h_2 vertical line and p_2 horizontal line. Read: $T_2 = +47$ °C and specific volume $v_2 = 29$ dm³/kg = 0.029 m³/kg.

STATE 3 – Saturated liquid condition already evaluated as saturation pressure $p_3 = 0.7698$ MPa. Read on *Table A.4.1*: enthalpy $h_3 = h_f = 427$ kJ/kg, $s_3 = s_f = 1.1431$ kJ/(kg·K) and specific volume $v_3 = v_f = 0.842$ dm³/kg.

STATE 4 – Enthalpy h_4 is equal to $h_3 = 427 \text{ kJ/kg}$, because of the isenthalpic throttling process, while the saturation pressure $p_4 = 0.133 \text{ MPa}$, at the evaporation temperature of $-20 \text{ }^\circ\text{C}$, has been already determined in state 1. On p - h diagram, point 4 is located at the intersection of isenthalpic vertical line with the isobaric horizontal line. Read: liquid-vapor quality $x_4 = 0.32$.

- b) Having determined the heat q_L extracted from the food store and the heat q_H withdrawn from the condenser, the coefficient of performance $COP_F = 3.7$ is obtained. This means that the energy extracted for the food store is 3.7 times the energy required by the compressor. The heat q_H can be verified as the sum of q_L and w .

$$39-2: q_L = h_1 - h_4 = 390 \text{ kJ/kg} - 241.61 \text{ kJ/kg} = 148.4 \text{ kJ/kg} \quad \blacktriangleleft$$

$$39-5: q_H = h_2 - h_3 = 430.2 \text{ kJ/kg} - 241.61 \text{ kJ/kg} = 188.6 \text{ kJ/kg} \quad \blacktriangleleft$$

$$39-1: COP_F = \frac{q_L}{w} = \frac{148.4 \text{ kJ/kg}}{40.2 \text{ kJ/kg}} = 3.7 \quad \blacktriangleleft$$

$$39-4: q_H = q_L + w = 148.4 \text{ kJ/kg} + 40.2 \text{ kJ/kg} = 188.6 \text{ kJ/kg}$$

- c) Being assigned the heat power $\dot{Q}_L = 100 \text{ kW}$, the refrigerant mass flow rate \dot{m} is obtained. Then the volume flow rate \dot{V}_1 is calculated. Being assigned $n = 1200 \text{ rpm} = 20 \text{ rps}$ (revolutions per second) and the volumetric efficiency $\eta_v = 0.75$, the total displacement $V_t = zV$ ($z = 6$ cylinders and V cylinder displacement) and the power P_a absorbed by the reciprocating compressor are obtained.

$$39-6: \dot{Q} = \dot{m}q \Rightarrow \dot{m} = \frac{\dot{Q}}{q} = \frac{100 \text{ kW}}{148.4 \text{ kJ/kg}} = 0.67 \frac{\text{kg}}{\text{s}} \quad \blacktriangleleft$$

$$39-7: \dot{V}_1 = \dot{m}v_1 = 0.67 \frac{\text{kg}}{\text{s}} \times 0.15 \frac{\text{m}^3}{\text{kg}} = 0.1 \frac{\text{m}^3}{\text{s}} \quad \blacktriangleleft$$

$$V_t = \frac{\dot{V}_1}{\eta_v n} = \frac{0.1 \text{ m}^3/\text{s}}{0.75 \times 20 \text{ rps}} = 0.0067 \text{ m}^3 = 6.7 \text{ dm}^3 \quad \blacktriangleleft$$

$$V_t = zV \Rightarrow V = \frac{V_t}{z} = \frac{6.7 \text{ dm}^3}{6} = 1.12 \text{ dm}^3$$

$$39-6: P_a = \frac{P_i}{\eta_m} = \frac{\dot{m}w_i}{\eta_m} = \frac{0.67 \text{ kg/s} \times 40.2 \text{ kJ/kg}}{0.95} = 28.5 \text{ kW} \quad \blacktriangleleft$$

COMMENT

1. Being a reciprocating compressor, its internal efficiency $\eta_c = 0.92$ is rather high. In case of a centrifugal compressor, the efficiency would be 0.8. With a refrigerant volume flow rate $\dot{V}_1 = 0.1 \text{ m}^3/\text{s}$, it is possible to adopt a reciprocating compressor. For $\dot{V}_1 \geq 0.25 \text{ m}^3/\text{s}$, the utilization of a less expensive centrifugal compressor becomes interesting.

- If all actual cycle losses are considered, and, in particular, the pressure drop in piping, the coefficient of performance $COP_F = 3.7$ becomes lower.
- This system can be used as a heat pump to warm up a house at $20\text{ }^\circ\text{C}$ by pumping heat in it. On a cold day ($-15\text{ }^\circ\text{C}$ outdoor) the heat pump would deliver to the house (39-6) a heat power $\dot{Q}_H = \dot{m}q_H = 0.67\text{ kg/s} \times 188.6\text{ kJ/kg} = 126.4\text{ kW}$, that is indeed comparable with a household gas-fired furnace. However, the owner will only have to pay for the electric power P_a absorbed to run the compressor, which in this case amounts to only 28.5 kW . The euro savings, over a few cold winters, could well offset the high capital outlay required to purchase and maintain the heat pump equipment.

39.5 Transcritical carbon dioxide systems

In a common refrigeration system, the refrigerant undergoes phase changes from vapor to liquid and from liquid back to vapor via superheating and subcooling phenomena, mainly because vapor must be cooled to become liquid and liquid must be heated back to become vapor (*Figure 39.5-a*). In transcritical systems, the refrigerant goes through another state of matter during the cycle. A *transcritical cycle* is a thermodynamic cycle where the working fluid goes through both subcritical and supercritical states. This is the case of carbon dioxide (CO_2) operating along a transcritical cycle, with its high pressure part above the critical pressure of 7.384 MPa . Such a value is the pressure (*Paragraph 16.2*) beyond which it is not entirely possible to talk about condensation temperature and subcooling, because the distinction between the two phases liquid and vapor vanishes, while below the critical point the two phases can coexist. Thus, CO_2 does not go through a regular condensation process but instead passes through a gas cooler in a supercritical phase (*Figure 39.8*).

Three steps characterize the process that utilizes the refrigerant R744 (CO_2) as a working fluid (*Figure 39.8-a*):

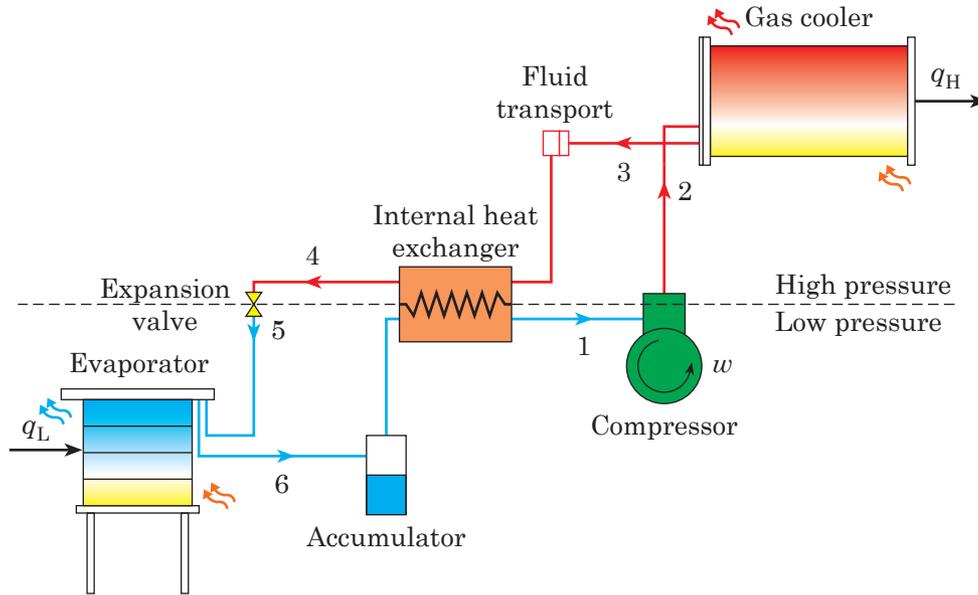
- the liquid carbon dioxide enters a first heat exchanger and then is vaporized to extract the heat q_L from the given space;
- a compressor supplies the work w , needed to increase pressure and temperature of CO_2 vapor;
- in the upper part of the circuit a second heat exchanger releases the heat q_H to the surroundings.

Figure 39.8 shows technical details of the CO_2 circuit and cycle:

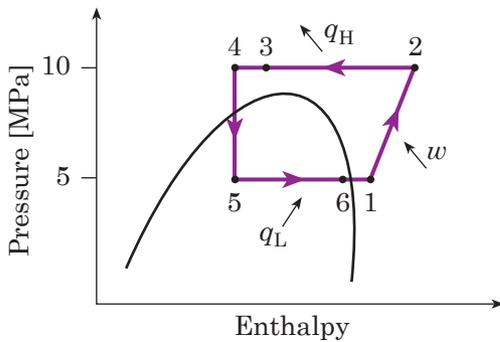
- the compressor draws superheated low-pressure vapor at point 1 from the internal heat exchanger. Vapor is generated by heat absorption in the evaporator and the internal heat exchanger;
- after increasing the pressure and temperature to point 2, the compressor discharges high pressure gas into the gas cooler;
- here, the supercritical CO_2 gas is cooled to point 3. Condensation does not occur since the pressure is supercritical. So that, the gas is cooled, and its density increased;
- from point 3 the fluid is further cooled to point 4 in the internal heat exchanger, giving off heat to the low-pressure vapor;

5. from point 4 the fluid is throttled to low pressure, giving a liquid-vapor mixture at the evaporator inlet (point 5);
6. liquid is vaporized due to heat absorption in the evaporator, giving a saturated or slightly wet vapor outlet at point 6.

(a)



(b)



(c)

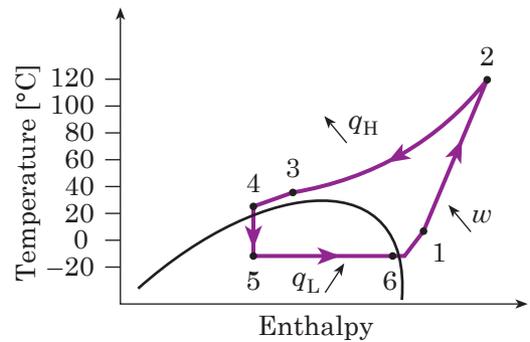


Fig. 39.8 - Cycle with carbon dioxide (CO_2) as working fluid both for refrigeration and heat-pump application (Shecco):

- a) The flow circuit is separated into a low pressure part and a high pressure part.
- b) The transcritical cycle in the pressure-enthalpy diagram. Since the high side pressure is supercritical, heat release takes place above the saturation dome. High side pressure is determined by the relationship between temperature, refrigerant charge, and volume, in the high side of the circuit. The change of high side pressure implies variation of cooling/heating capacity and coefficient of performance. The diagram also shows how the constant enthalpy throttling into the two phase regime forms a liquid and vapor phase.
- c) The temperature-enthalpy diagram shows how heat is rejected from the transcritical cycle at gliding temperature; the refrigerant temperature profile can be adapted to heating of water and/or air to a high temperature.

39.6 Absorption cycle

The absorption cycle is similar to the vapor compression cycle, with one major change: the compressor of a conventional vapor refrigeration system is replaced by an absorber-generator-pump assembly, noted through the square gray line box in *Figure 39.9*, with thermal energy replacing the compressor mechanical energy to compress the refrigerant. Two working fluids are required: a refrigerant and the other one absorbent. The system working fluid, shown here, is ammonia as refrigerant and water as absorbent; another interesting fluid is composed by lithium bromide (a solid salt) as the absorbent and water as refrigerant. The main characteristic of these couples of fluids is to generate an exothermic mixing process and then an endothermic separation process. If one of two fluids is a refrigerant, it is possible to imagine a refrigeration cycle, composed by a series of processes typical of reversed cycle (condensation, throttling, evaporation) actuated by the pure refrigerant, plus a mixing with an absorbent (from this, the name of *absorption cycle*) and subsequent separation made at the expenses of external thermal energy. Being the working fluid compressed as a liquid, the work is negligible.

As in the vapor-compression cycle, the absorption cycle starts with the refrigerant that, passing through the evaporator, removes the heat q_L from the low-temperature region. The low-pressure ammonia vapor leaving the evaporator enters the absorber where it is absorbed in the weak ammonia solution through a process releasing the heat q'_L (the temperature is slightly above the surroundings). The strong ammonia solution is then pumped via a heat exchanger to the generator where higher pressure and temperature conditions are maintained by means of heat q'_H supplied by a high temperature source. As in the vapor-compression cycle, the ammonia vapor goes to the condenser where, it is condensed and then to the expansion valve and the evaporator. Although pumping a liquid is an advantage on

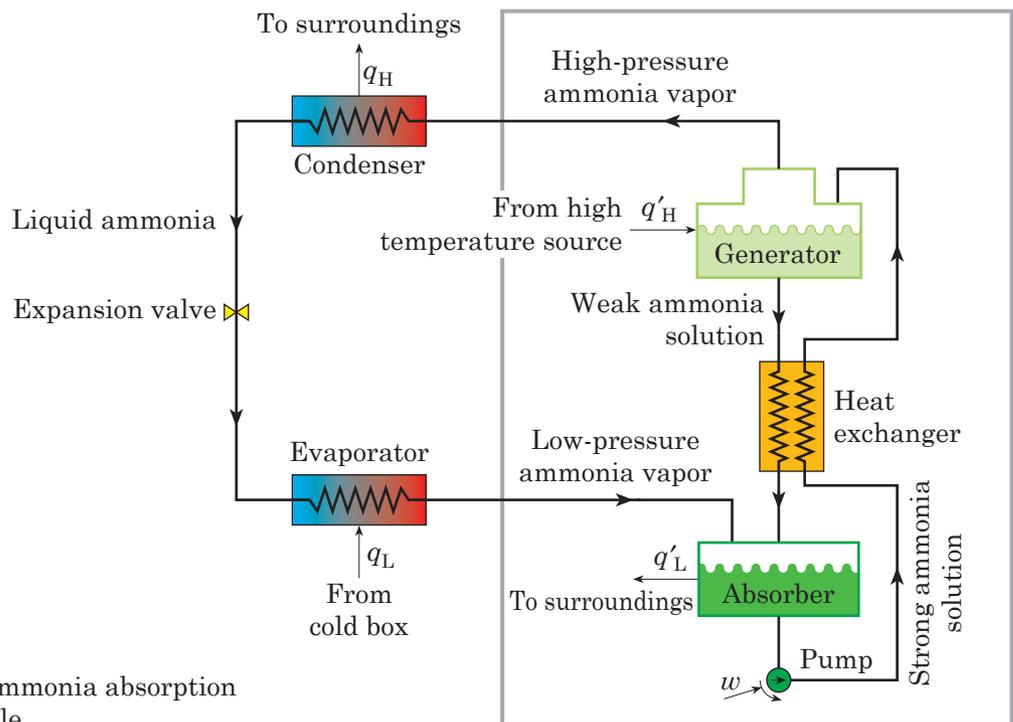


Fig. 39.9 - The ammonia absorption refrigeration cycle.

vapor compression, this cycle requires a much larger heat-rejection system. The absorption cycle may be an economically attractive method to provide refrigeration in industrial processes where waste heat (at least higher than $100 + 150\text{ }^{\circ}\text{C}$) is available as a byproduct.

39.7 Air refrigeration cycle

The *air refrigeration cycle* is also known as the Brayton cooling cycle because it is essentially a *reversed Brayton turbine cycle*. It is not common due to its low coefficient of performance (see *Example 39.3*). However, being air non-flammable, readily available and non-toxic, the air refrigeration cycle is in common use aboard jet aircraft and gas liquefaction application where a refrigerant with such characteristics is needed. *Figure 39.10* shows the air refrigeration cycle where compressor and expander are usually dynamic machines.

After compression $1 \Rightarrow 2$, the air is cooled as a result of the heat transfer q_H to the surroundings. The air is then expanded in process $3 \Rightarrow 4$ to the pressure p_4 entering the compressor, while the temperature drops to T_4 and the heat q_L is transferred to the air until temperature T_L is reached. The work w absorbed by area 1-2-3-4-1 of the ideal cycle of *Figure 30.10*, while the refrigeration effect is represented by area 4-1-B-A-4; therefore, the coefficient of performance is the ratio of these two areas. If the air is considered an ideal gas (*Table A.5*), then the ideal gas equations of *Appendix A.6* can be used to find the properties at each point of the cycle.

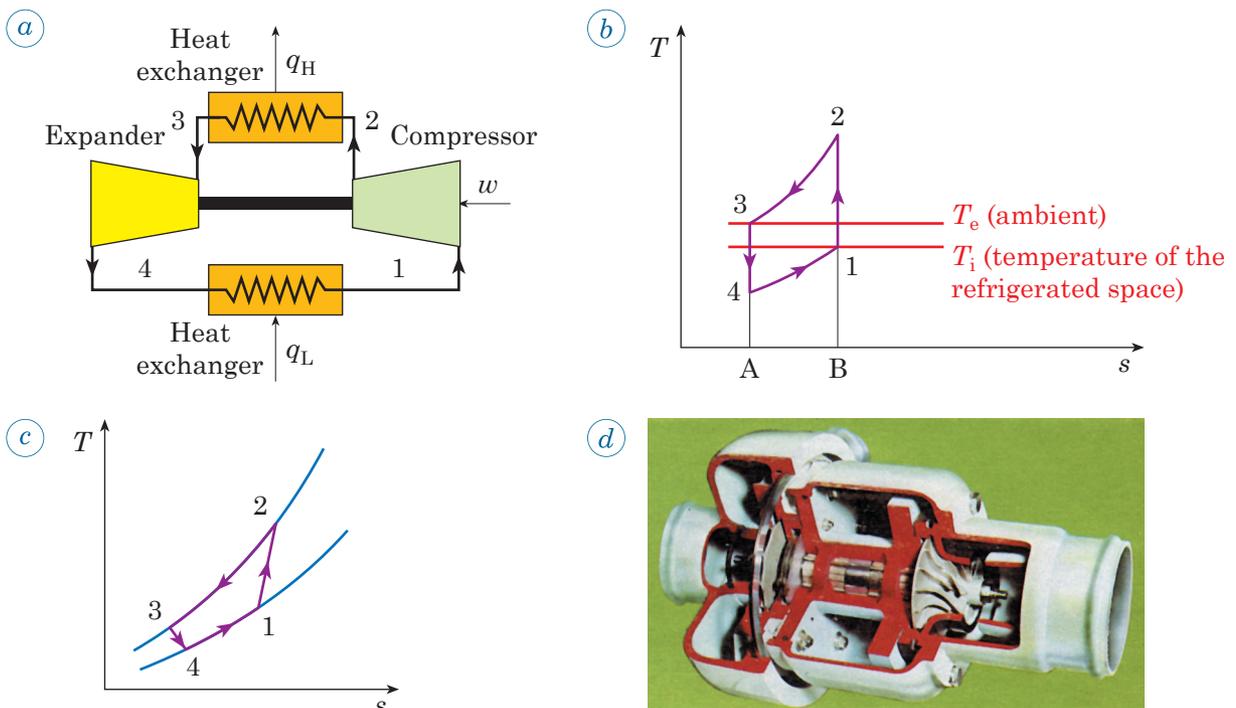


Fig. 39.10 - The air refrigeration cycle:

- a) the system;
- b) ideal cycle;

- c) actual cycle with diagonal compression ($1 \Rightarrow 2$) and expansion line ($3 \Rightarrow 4$);
- d) air conditioning in aircrafts (Garrett).

Example 39.3 Air compression cycle

With reference to ideal air refrigeration cycle of *Figure 39.10-b*, air enters the compressor at atmospheric pressure ($p_1 = p_4 = 0.1 \text{ MPa}$) and temperature ($T_1 = 253 \text{ K} \approx -20 \text{ }^\circ\text{C}$), and leaves at $p_2 = p_3 = 0.6 \text{ MPa}$. Air enters the expander at $T_3 = 285 \text{ K} (\approx 12 \text{ }^\circ\text{C})$. Determine:

- the work w absorbed by the cycle;
- the coefficient of performance of the refrigeration cycle COP_F and the air mass flow rate \dot{m} required for a refrigeration power $\dot{Q}_L = 15 \text{ kW}$.

SOLUTION

- Being $1 \Rightarrow 2$ an isentropic compression and $3 \Rightarrow 4$ an isentropic expansion, the cycle temperatures are determined by taking (*Table A.5*) a constant pressure specific heat of the air $c_p = 1.0035 \text{ kJ}/(\text{kg}\cdot\text{K})$ and the ratio of specific heats $\gamma = 1.4$. Then the cycle work w is calculated (*Paragraphs 30.3* and *38.3.2*) as a difference of the work w_c required by turbo-compressor and work w_t due to the turbo-expander.

$$\mathbf{A-39:} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} = \left(\frac{0.6 \text{ MPa}}{0.1 \text{ MPa}} \right)^{(1.4-1)/1.4} = 6^{0.286} = 1.669 \Rightarrow$$

$$\Rightarrow T_2 = 1.669T_1 = 1.669 \times 253 \text{ K} = 422 \text{ K}$$

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4} \right)^{(\gamma-1)/\gamma} = \left(\frac{0.6 \text{ MPa}}{0.1 \text{ MPa}} \right)^{(1.4-1)/1.4} = 1.669 \Rightarrow T_4 = \frac{T_3}{1.669} = \frac{285 \text{ K}}{1.669} = 171 \text{ K}$$

$$\mathbf{31-5, 28-10:} \quad w_c = h_2 - h_1 = c_p(T_2 - T_1) = 1.0035 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} (422 - 253) \text{ K} = 169.6 \frac{\text{kJ}}{\text{kg}}$$

$$w_t = h_3 - h_4 = c_p(T_3 - T_4) = 1.0035 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} (285 - 171) \text{ K} = 114.4 \frac{\text{kJ}}{\text{kg}}$$

$$w = w_c - w_t = 169.6 \frac{\text{kJ}}{\text{kg}} - 114.40 \frac{\text{kJ}}{\text{kg}} = 55.2 \frac{\text{kJ}}{\text{kg}} \quad \blacktriangleleft$$

- At first determine the heat quantities q_H and q_L ; then calculate the coefficient of performance of the refrigeration cycle COP_F and the air mass flow rate \dot{m} required for a refrigeration power $\dot{Q}_L = 15 \text{ kW}$.

$$\mathbf{31-6, 28-10:} \quad q_H = h_2 - h_3 = c_p(T_2 - T_3) = 1.0035 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} (422 - 285) \text{ K} = 137.5 \frac{\text{kJ}}{\text{kg}}$$

$$q_L = h_1 - h_4 = c_p(T_1 - T_4) = 1.0035 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} (253 - 171) \text{ K} = 82.3 \frac{\text{kJ}}{\text{kg}}$$

$$\mathbf{39-4:} \quad q_H = q_L + w = 82.3 \text{ kJ/kg} + 55.2 \text{ kJ/kg} = 137.5 \text{ kJ/kg} \quad (\text{for validation})$$

$$39-1: COP_F = \frac{q_L}{w} = \frac{82.3 \text{ kJ/kg}}{55.2 \text{ kJ/kg}} = 1.5$$

$$39-6: \dot{Q}_L = \dot{m}q_L \Rightarrow \dot{m} = \frac{\dot{Q}_L}{q_L} = \frac{15 \text{ kW}}{82.3 \text{ kJ/kg}} = 0.18 \frac{\text{kg}}{\text{s}}$$

COMMENT The coefficient of performance of the air refrigeration cycle, even evaluated on the ideal cycle, is sensibly lower than that of vapor-compression cycle. The low value of coefficient of performance has restricted the use of the air refrigeration cycle to applications mentioned at the very beginning of *Paragraph 39.7*.

39.8 Heat pump

If we reverse the three arrows of the heat engine shown in *Figure 30.3-a*, we obtain a device that absorbs work (*Figure 30.4-a*). It would take heat in (q_L) from a low-temperature (T_L) source and reject heat (q_H) to a high temperature (T_H) sink. Such a device is called a *heat pump*, because it moves energy as heat from a low- to a high-temperature region, somewhat like a pump would move water from a low- to a high-elevation region. In a thermodynamic sense, heat pump cycles are identical to refrigeration cycles, the major difference being the desired effect (*Figure 39.1*): in the heating cycle, heat is extracted from a natural or waste heat source and transferred to the interested space (*Figure 39.11*); in the cooling cycle, heat is removed from the interested space and discharged to a heat sink (*Figure 39.3*). Thus, the same unit can provide both heating in the winter and cooling in the summer.

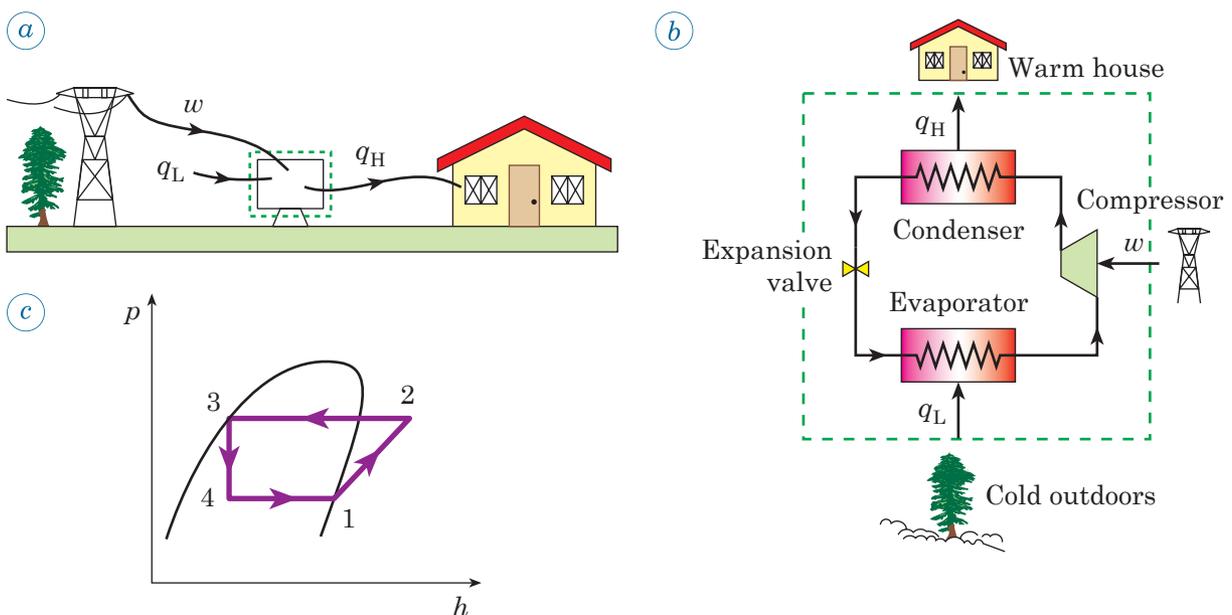


Fig. 39.11 - The heat pump:
a) the picture; b) the system; c) the process representation.

The *coefficient of performance* of the heat pump COP_P (30-2) is always higher than 1 and it is equal to the coefficient of performance of the refrigeration cycle COP_F increased by 1, according to the equation 39-4 ($q_H = q_L + w$) we have:

$$COP_P = \frac{q_H}{w} \quad 39-8$$

$$COP_P = \frac{q_H}{w} = \frac{q_L + w}{w} = 1 + \frac{q_L}{w} = 1 + COP_F \quad 39-8'$$

The heat pumps differ in the type of heat exchangers (depending on the heat sink and source), and working fluid. There are four basic types of heat pumps: air-to-air, water-to-air, water-to-water, and earth-to-air. For instance, heat pumps used in the small window conditioners are of air-to-air type, while in large buildings they are of water-to-water type. As in the *Example 39.4*, the low-temperature source is a lake cold water, while the heat sink is the warm water of the heating system.

The heat pumps operated by an internal combustion engine are of particular interest because of very high efficiency that can be reached by combining the heat recovered from the exhaust gases with that supplied by the heat pump.

Example 39.4 Heat pump

It is required to supply heat to a building at the rate of 1500 kW with warm water at 45 °C. The cold water of a lake, that serves as a source for the heat pump, is at 14 °C. By taking into account the necessary temperature difference between the working fluid (R134a) and the two areas interesting the heat transfer (the first area is the warm water of the heating, while the second area is the cold water source), the temperature in the condenser is 60 °C and the temperature in the evaporator is 5 °C. The refrigerant, as a saturated vapor, enters a centrifugal compressor having $\eta_c = 0.8$ as internal efficiency. Determine:

- the coefficient of performance COP_P of the heat pump;
- the mass flow rate \dot{m} of the refrigerant;
- the water mass flow rate \dot{m}_w , knowing that water enters the heating system at 45 °C (after heat exchanged in the condenser) and leaves at 35 °C;
- the power absorbed by compressor with a mechanical efficiency $\eta_m = 0.98$.

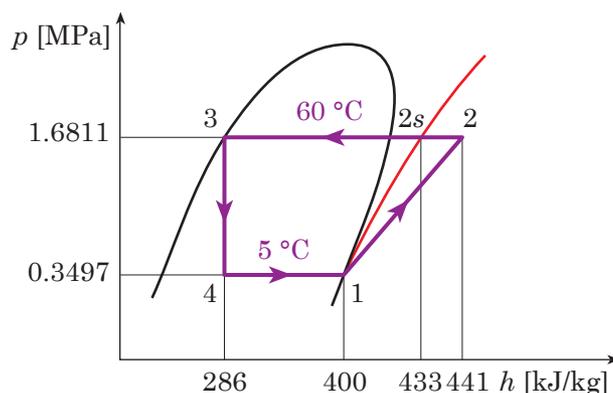


Fig. 39.12 - Heat pump cycle of *Example 39.4*.

State ^(*)	T [°C]	p [MPa]	h [kJ/kg]	s [kJ/kg·K]	ν [dm ³ /kg]	Fluid [R134a]
1	5	0.3497	400.44	1.7207	58.11	Saturated vapor
2s	65	1.6811	433	1.7207	12.30	Superheated vapor
2	72	1.6811	441.2	—	12.50	Superheated vapor
3	60	1.6811	286.53	1.2817	0.949	Saturated liquid
4	5	0.3497	286.53	—	—	Liquid-vapor ($x = 0.37$)

(*) Red values are obtained from p - h diagram, while black values are read on *Table A.4*.

SOLUTION

a) *Figure 39.12* shows the properties of each point of the cycle.

STATE 1 – For the evaporation temperature of +5 °C, read on *Table A.4.1* saturation pressure $p_1 = 0.3497$ MPa and other quantities $h_1 = h_g$, $s_1 = s_g$ and $\nu_1 = \nu_g$ quoted in the above Table.

STATE 2s – Being an isentropic compression, $s_{2s} = s_1 = 1.7207$ kJ/(kg·K). Vapor pressure $p_{2s} = p_2$ equals $p_3 = 1.6811$ MPa, that is the saturation pressure corresponding to the condensation temperature $T_3 = +60$ °C. Then moving on p - h diagram along the isentropic orange line from 1 until the horizontal line with pressure $p_3 = p_{2s}$, point 2s is located. Read: $T_{2s} = +65$ °C and other quantities h_{2s} and ν_{2s} quoted in Table.

STATE 2 – At first the ideal compression work w_s is calculated. Taking into account the internal compression efficiency $\eta_c = 0.8$, the internal work w_i and then the actual enthalpy h_2 are obtained.

$$\mathbf{39-3:} \quad w_s = h_{2s} - h_1 = 433 \text{ kJ/kg} - 400.44 \text{ kJ/kg} = 32.6 \text{ kJ/kg}$$

$$\mathbf{31-8:} \quad \eta_c = \frac{w_s}{w_i} \Rightarrow w_i = \frac{w_s}{\eta_c} = \frac{32.6 \text{ kJ/kg}}{0.8} = 40.75 \frac{\text{kJ}}{\text{kg}}$$

$$\mathbf{39-3:} \quad w_i = h_2 - h_1 \Rightarrow h_2 = w_i + h_1 = 40.75 \text{ kJ/kg} + 400.44 \text{ kJ/kg} = 441.2 \text{ kJ/kg}$$

The point 2, that represents the fluid condition leaving the compressor, is found as intersection, on p - h diagram, between h_2 vertical line and p_2 horizontal line. Read: $T_2 = +72$ °C and specific volume $\nu_2 = 12.5$ dm³/kg.

STATE 3 – Saturated liquid condition already evaluated as saturation pressure p_3 . Read on *Table A.4.1*: enthalpy $h_3 = h_f$, entropy $s_3 = s_f$ and specific volume $\nu_3 = \nu_f$.

STATE 4 – Enthalpy h_4 is equal to $h_3 = 286.53$ kJ/kg, because of the isenthalpic throttling process, while the saturation pressure $p_4 = 0.3497$ MPa, at the evaporation temperature of +5 °C, has been already determined in state 1. On p - h diagram, point 4 is located at the intersection of isenthalpic vertical line with the isobaric horizontal line. Read: liquid-vapor quality $x_4 = 0.41$.

$$\mathbf{39-5:} \quad q_H = h_2 - h_3 = 441.2 \text{ kJ/kg} - 286.53 \text{ kJ/kg} = 154.7 \text{ kJ/kg}$$

$$\mathbf{39-8:} \quad COP_p = \frac{q_H}{w} = \frac{154.7 \text{ kJ/kg}}{40.75 \text{ kJ/kg}} = 3.8 \quad \blacktriangleleft$$

b) The heat power to be supplied to the heating system is assigned: $\dot{Q}_H = 1500 \text{ kW} = 1500 \text{ kJ/s}$.

$$\mathbf{39-6:} \quad \dot{Q}_H = \dot{m}q_H \Rightarrow \dot{m} = \frac{\dot{Q}_H}{q_H} = \frac{1500 \text{ kJ/s}}{154.7 \text{ kJ/kg}} = 9.7 \frac{\text{kg}}{\text{s}} \quad \blacktriangleleft$$

c) The heat power \dot{Q}_H is supplied to the water mass flow rate \dot{m}_w in such a way that after being circulated in the system until 35°C , can achieve again 45°C . Then, \dot{Q}_H is equal to the product of \dot{m}_w for the water specific heat $c_w = 4.186 \text{ kJ/(kg}\cdot\text{K)}$ and the temperature difference $\Delta T = 45^\circ\text{C} - 35^\circ\text{C} = 10^\circ\text{C}$.

$$\dot{Q}_H = \dot{m}_w c_w \Delta T \Rightarrow \dot{m}_w = \frac{\dot{Q}_H}{c_w \Delta T} = \frac{1500 \text{ kJ/s}}{4.186 \text{ kJ/(kg}\cdot\text{K)} \times 10 \text{ K}} = 35.83 \frac{\text{kg}}{\text{s}} \quad \blacktriangleleft$$

d) The power absorbed by compressor having a mechanical efficiency $\eta_m = 0.98$ is:

$$\mathbf{39-6:} \quad P_a = \frac{P_i}{\eta_m} = \frac{\dot{m}w_i}{\eta_m} = \frac{9.7 \text{ kg/s} \times 40.75 \text{ kJ/kg}}{0.98} = 403.34 \text{ kW} \quad \blacktriangleleft$$

COMMENT The heat q_L extracted by the R134a from cold water during the evaporation process is:

$$\mathbf{39-2:} \quad q_L = h_1 - h_4 = 400.44 \text{ kJ/kg} - 286.53 \text{ kJ/kg} = 113.91 \text{ kJ/kg}$$

The condensation heat q_H supplied to the heating circuit:

$$\mathbf{39-4:} \quad q_L + w = q_H \Rightarrow 113.91 \text{ kJ/kg} + 40.75 \text{ kJ/kg} = 154.7 \text{ kJ/kg}$$

points out that, of the total heat required $q_H = 154.7 \text{ kJ/kg}$, only 40.75 kJ/kg is absorbed by compressor, while the major amount $q_L = 113.91 \text{ kJ/kg}$ is supplied by the cold water.

/// 39.9 Air and vapor mixtures

Atmospheric air contains small amounts of moisture (it is common to refer to the atmosphere as *moist air*) and can be considered to be a mixture of two ideal gases, dry air (*Table 32.1*) and water vapor. From Dalton's law, the total atmospheric pressure p [Pa] is the sum of dry air partial pressure p_{da} and water vapor pressure p_w :

$$p = p_{da} + p_w \quad \mathbf{39-9}$$

The study of the properties and behavior of the moist air, known as *psychrometrics*, has to be preceded by three definitions of temperature [$^\circ\text{C}$]:

- *dry-bulb temperature*, T – this is the equilibrium temperature that a regular thermometer measures if exposed to the atmospheric air;
- *dew-point temperature*, T_d – this is the dry-bulb temperature at which water vapor starts to condense out when moist air is cooled in a constant pressure process;

- *thermodynamic wet-bulb temperature*^{39.5}, T^* – this is the air temperature measured by means of a thermometer with a bulb covered by a wick that has been thoroughly wetted with pure water.

Suppose that (T - s diagram for the vapor of *Figure 39.13*) the temperature of the gaseous mixture and the partial pressure of the vapor in the mixture are such that the vapor is initially superheated at state A. If the mixture is cooled at constant pressure, the partial pressure of the vapor remains constant until point B, that lies on the saturated-vapor line, and then condensation begins. The temperature at state B is the dew-point temperature T_d . If, for example, an air stream licks a cooled pipe, the formation of water droplets shows that the surface temperature is below the dew point of the vapor in the moist air touching the pipe. We can also define the dew-point as the saturation temperature that corresponds to the partial pressure p_w of water vapor in the air. The temperature at A of the air-vapor mixture is the actual temperature of moist air, i.e. the dry-bulb temperature. The vapor saturation condition at dry-bulb temperature is that at C; pressure at C is the pressure of saturated pure water that corresponds to the dry-bulb temperature and it is indicated with p_{ws} .

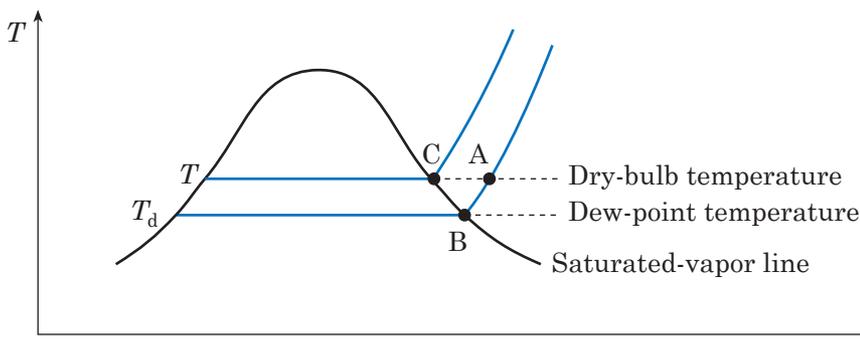


Fig. 39.13 - Temperature-entropy diagram to show definition of the dew point.

In the adiabatic saturation process an air-vapor mixture comes in contact with a body of water (*Figure 39.14-a*) or water sprays in a well-insulated duct (*Figure 39.14-b*): moisture is supplied by the evaporation of liquid water without any external energy source. If the mixture leaving the duct is saturated and if the process is adiabatic, the temperature of the mixture is known as the adiabatic saturation temperature; it is a thermodynamic property^{39.5} of the moist air at the inlet state.

39.5 - For any state of moist air, a temperature T^* exists at which liquid (or solid water) evaporates into the air to bring it to saturation at exactly this same temperature and total pressure: during adiabatic saturation, saturated air is expelled at a temperature equal to that of the injected water. Thus the thermodynamic wet-bulb temperature is a thermodynamic property of a mixture of air and water vapor. The value indicated by a simple wet-bulb thermometer often provides an adequate approximation of the thermodynamic wet-bulb temperature. For an accurate wet-bulb thermometer, “the wet-bulb temperature” and the adiabatic saturation temperature are approximately equal for air-water vapor mixtures at atmospheric temperature and pressure.

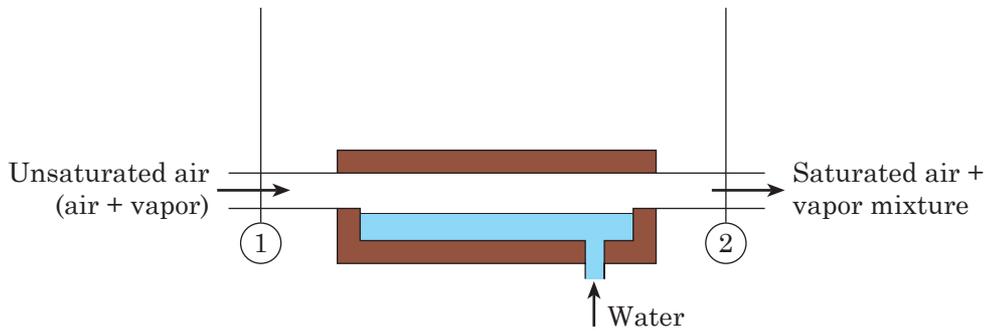


Fig. 39.14-a - The adiabatic saturation process.

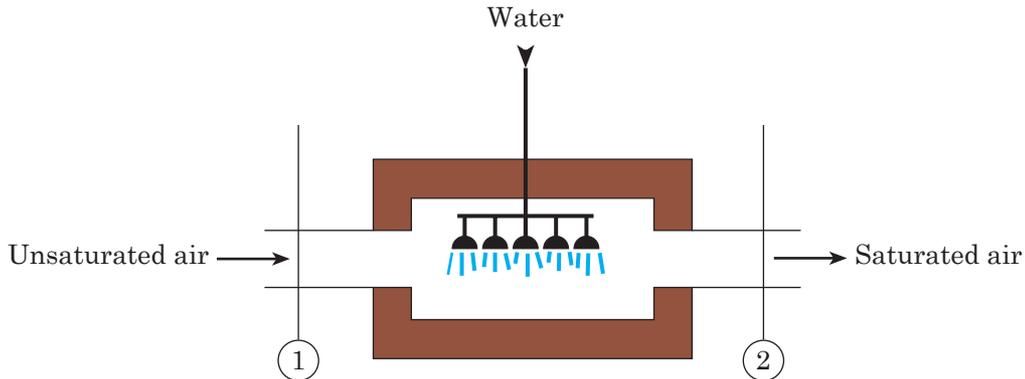


Fig. 39.14-b - Atmospheric air enters the adiabatic steady-flow system at (1) and liquid water is sprayed into the air stream in such a manner that the air leaving at (2) is saturated.

The moisture of the air-vapor mixture is determined with a psychrometer (*Figure 39.15*). This consists of two thermometers; one thermometer's bulb is covered by a wick that has been thoroughly wetted with water. When the wet bulb is placed in an airstream, water evaporates from the wick, measuring the wet-bulb temperature T^* . The other thermometer simply measures the air temperature T . If the space is not saturated, water on the wick evaporates and, extracting heat, cools down the bulb reaching a temperature lower than the ambient temperature. The more moisture the air contains, the higher is the temperature difference between the two thermometers. On the contrary, if the air-vapor mixture is saturated, the pure water on the wick is not able to evaporate and the two thermometers show the same temperature.

For every temperature, there is a unique vapor pressure, p_{ws} , which represents the maximum pressure the water vapor can exert. The actual vapor pressure, p_w , can be less than or equal to, but not greater than the saturation value. The saturation pressure is found from steam tables as the pressure corresponding to the dry-bulb temperature of the atmospheric air. If the vapor pressure equals the saturation pressure, the air is said to be saturated. Saturated air is a mixture of dry air and saturated water vapor; when the air is saturated, all three temperatures – dew-point T_d , wet-bulb T^* and dry-bulb T – are equal: $T_d = T^* = T|_{\text{sat}}$. Unsaturated air is a mixture of dry air and superheated water vapor; when the air is unsaturated, the dew-point temperature T_d will be less than the wet-bulb temperature T^* : $T_d < T^* < T|_{\text{unsat}}$.

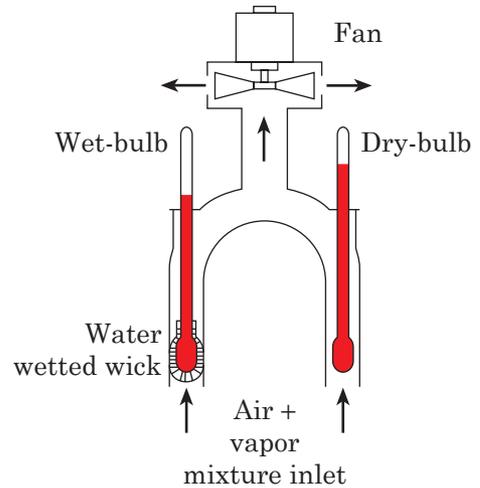


Fig. 39.15 - A psychrometer consists of two thermometers: one thermometer's bulb is covered by a wick that has been thoroughly wetted with pure water. When the wet bulb is placed in an airstream, water evaporates from the wick, eventually reaching an equilibrium temperature called the wet-bulb temperature.

39.10 Properties of the atmospheric air

The *humidity ratio* of the air-vapor mixture $x^{39.6}$ [kg_w/kg_{da}] is the mass ratio of water vapor mass m_w [kg_w] to dry air m_{da} [kg_{da}]; the humidity ratio can be written also in terms of mass flow rate [kg/s] of water vapor \dot{m}_w and dry air \dot{m}_{da} . As a consequence, the mass m or the mass flow rate \dot{m} of the moist air will be the sum of two masses or two mass flow rates of vapor and dry air.

$$x = \frac{m_w}{m_{da}} = \frac{\dot{m}_w}{\dot{m}_{da}} \quad \dot{m}_w = x \cdot \dot{m}_{da} \quad \dot{m} = \dot{m}_w + \dot{m}_{da} = (1 + x) \cdot \dot{m}_{da} \quad \mathbf{39-10}$$

By taking into account water and air molal masses M [kg/kmol] (*Table A.5*) and from the equation of state for an ideal gas (*Paragraph 32.2*), water vapor mass m_w and dry air mass m_{da} in the volume V [m³] are obtained as a function of their partial pressures p_w and p_{da} . Since the dry air partial pressure p_{da} can be substituted (**39-9**) by $p - p_w$, partial pressures p_{da} and p_w can be written versus humidity ratio x and ambient pressure p ; as a consequence in a process with a constant (total) pressure, p_{da} and p_w remain separately constant until the humidity ratio x remains constant.

$$x = \frac{m_w}{m_{da}} = \frac{p_w VM_w}{p_{da} VM_{da}} = \frac{M_w}{M_{da}} \frac{p_w}{p_{da}} = \frac{18.015 \text{ kg/kmol}}{28.97 \text{ kg/kmol}} \frac{p_w}{p_{da}} = 0.622 \frac{p_w}{p_{da}} \Rightarrow$$

39.6 - Symbols of the main quantities of moist air are those used by ASHRAE (American Society of Heating, Refrigerating and Air conditioning Engineers). Exceptions are the rate of heat transfer represented by Q , universally adopted in scientific writings, and the humidity ratio for which is used the Italian symbol x instead of w used by ASHRAE. The symbol x recalls the quality of the liquid-vapor mixtures; on the contrary, the symbol w is universally adopted to indicate the mass fraction of a constituent of perfect-gas mixture (**32-11**), while x represents the volume fraction (**32-10**), as correctly indicated by ASHRAE.

$$x = 0.622 \frac{p_w}{p - p_w} \Rightarrow \begin{cases} p_{da} = \frac{0.622}{0.622 + x} p \\ p_w = \frac{x}{0.622 + x} p \end{cases} \quad \mathbf{39-11}$$

The relative humidity ϕ [-] is the ratio of the measured amount of moisture in the air, m_w or p_w , to the maximum amount of moisture the air can hold, m_{ws} or saturation vapor pressure of pure water p_{ws} , at the same temperature and pressure. Relative humidity is expressed in percent of saturation. Air with a relative humidity of 40, for example, is holding 40% of the moisture that it is capable of holding at that temperature and pressure.

$$\phi = \frac{m_w}{m_{ws}} = \frac{p_w}{p_{ws}} \quad \mathbf{39-12}$$

The pressure p_{ws} [Pa] can be approximated by means of the following formula^{39.7}:

$$p_{ws} \cong 1000 \cdot e^{\left(16.6536 - \frac{4030.183}{T + 235}\right)} \Rightarrow T = \frac{4030.183}{16.6536 - \ln(p_{ws}/1000)} - 235 \quad \ll T \text{ [}^\circ\text{C]} \gg \quad \mathbf{39-13}$$

while the equation **39-11** of the humidity ratio x , as a function of relative humidity ϕ and saturated vapor pressure p_{ws} , becomes:

$$x = 0.622 \frac{\phi p_{ws}}{p - \phi p_{ws}} \quad \mathbf{39-14}$$

or, if x and p_{ws} known, to obtain ϕ :
$$\phi = x \frac{(p - p_{ws})}{0.622 \cdot p_{ws}} \quad \mathbf{39-14'}$$

Enthalpy h [kJ/kg_{da}] of the moist air is the sum of dry air enthalpy h_{da} and saturated vapor enthalpy h_w : $h = h_{da} + xh_w$. By multiplying h_w [kJ/kg_w] for humidity ratio x [kg_w/kg_{da}], the vapor enthalpy referred to the dry air is obtained: xh_w [kJ/kg_{da}]. From several formulae approximating the dry air enthalpy h_{da} and saturated vapor (h_g in *Table A.3.1*), the ASHRAE formula^{39.8} is used: $h_{da} = c_{p,da} T \cong 1.006 \text{ kJ/(kg}\cdot^\circ\text{C)} \times T$ [°C], with $c_{p,da}$ constant-pressure specific heat of dry air, and $h_w \cong 2501 \text{ kJ/kg} + 1.86 \text{ kJ/(kg}\cdot^\circ\text{C)} \times T$ [°C], with

39.7 - Other formulae, different from **39-13**, can be used to calculate the saturation pressure p_{ws} such as, for example, two formulae, covering two temperature ranges, as follows:

$$p_{ws} = 611 \cdot e^{\left(82.9 \cdot 10^{-3} \cdot T - 288.1 \cdot 10^{-6} \cdot T^2 + 4.403 \cdot 10^{-6} \cdot T^3\right)} \quad \ll -30 \text{ }^\circ\text{C} \leq T < 0 \text{ }^\circ\text{C} \gg$$

$$p_{ws} = 611 \cdot e^{\left(72.5 \cdot 10^{-3} \cdot T - 288.1 \cdot 10^{-6} \cdot T^2 + 0.79 \cdot 10^{-6} \cdot T^3\right)} \quad \ll 0 \text{ }^\circ\text{C} \leq T < 40 \text{ }^\circ\text{C} \gg$$

39.8 - The formula of moist air enthalpy h is that of ASHRAE (2014 edition of psychrometrics). As an alternative, the following two formulae can be used:

$$h \cong 1.005 \cdot T + x \cdot (2501.3 + 1.82 \cdot T) \quad \text{or} \quad h \cong 1 \cdot T + x \cdot (2500 + 1.9 \cdot T)$$

2501 kJ/kg vaporization enthalpy of water at 0 °C (*Table A.3.1*) and 1,86 kJ/(kg_w·°C) vapor specific heat in the range (−10 °C) ÷ (+50 °C).

$$h = h_{da} + xh_w \cong 1.006 \cdot T + x(2501 + 1.86 \cdot T) \quad \mathbf{39-15}$$

or, if x and h known, to obtain T [°C]:
$$T = \frac{h - 2501 \cdot x}{1.006 + 1.86 \cdot x} \quad \mathbf{39-15'}$$

The specific volume ν [m³/kg_{da}], per mass of dry air m_{da} , is obtained from the ideal gas equation **28-3**, where temperature T is measured in kelvin and the dry air constant is (*Table A.5*) $R_{da} = 0.287$ kJ/(kg·K). Since **39-11**, the dry air pressure p_{da} is a function of total pressure p and humidity ratio x : $p_{da} = (0.622p)/(0.622 + x)$.

$$\nu = \frac{V}{m_{da}} = \frac{R_{da}T}{p_{da}} = \frac{R_{da}T}{0.622p}(0.622 + x) \quad \ll T \text{ [K]} \gg \quad \mathbf{39-16}$$

The air density ρ [kg/m³] is the mass of the moist air $m_{da} + m_w$ contained in the volume V ; then it is not equal to reciprocal of specific volume ν , being the last referred to dry air mass.

$$\rho = \frac{m_{da} + m_w}{V} = \frac{m_{da}}{V}(1 + x) = \frac{(1 + x)}{\nu} = \frac{0.622p}{R_{da}T} \left(\frac{1 + x}{0.622 + x} \right) \quad \mathbf{39-17}$$

39.11 The psychrometric chart

Instead of using the above mathematical relationships for the atmospheric air, psychrometric properties can be read directly from *psychrometric charts* (“psych charts”, as they are usually referred to) as illustrated in *Figure 39.16*. There are different psychrometric charts for low, medium, and high temperature ranges, as well as charts for different atmospheric pressure, i.e. elevations. Two charts can be considered: the *Mollier* diagram with oblique axes and coordinates represented by humidity ratio and enthalpy (*Figure 39.16-a*), and *Carrier’s* chart, that was the first to develop a chart derived from Mollier’s one and purposely designed for air conditioning.

The major lines and scales on the Carrier’s chart (*Figure 39.16-b*) are:

- Dry-bulb temperature purple lines, T [°C];
- Wet-bulb temperature black dotted lines, T^* [°C];
- Enthalpy or total heat orange lines, h [kJ/kg_{da}];
- Relative humidity black lines, ϕ [%];
- Humidity ratio or moisture content blue lines, x [g_w/kg_{da}];
- Saturation temperature or dew point black scale, T_d [°C];
- Specific volume green lines, ν [m³/kg_{da}].

The protractor to the left of the chart shows two scales:

- ratio of enthalpy difference to humidity ratio difference, Δh [kJ/kg_{da}] / Δx [g_w/kg_{da}];
- sensible/total heat ratio, Q_s [kJ] / Q_t [kJ].

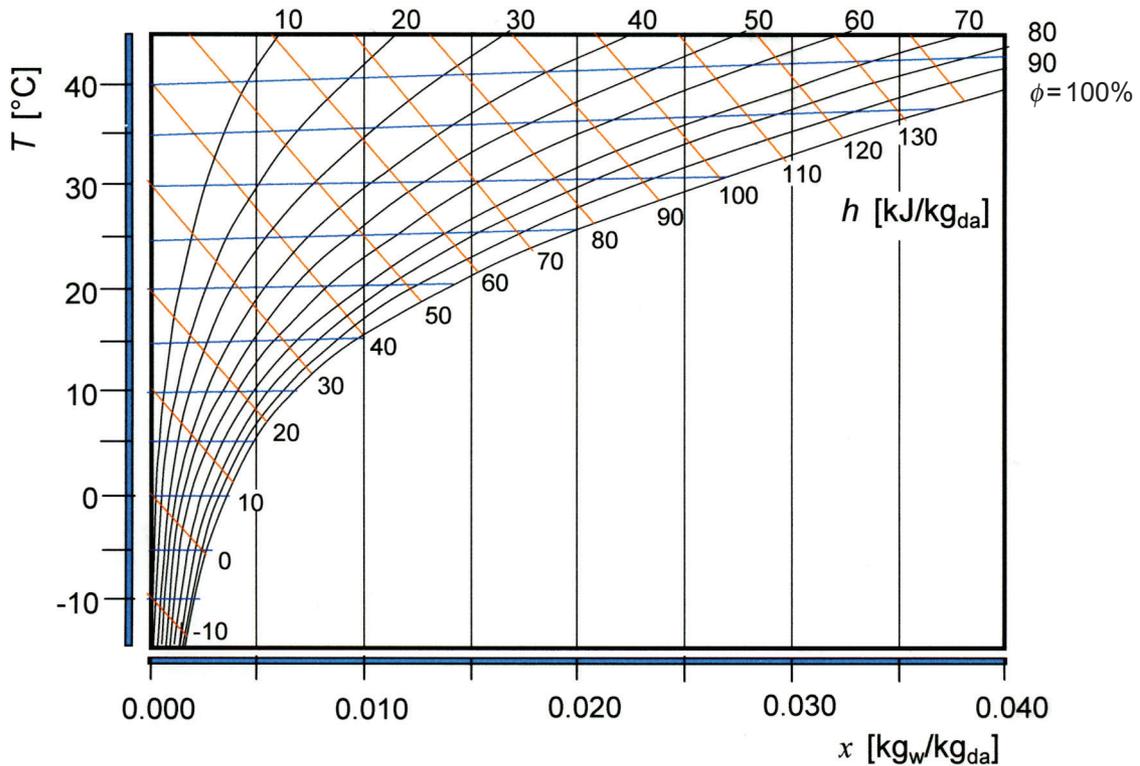


Fig. 39.16-a - Mollier psychrometric chart.

Sensible heat Q_s is the heat that changes the temperature of the air without changing its moisture content. Heat added to air by a heating coil is an example of sensible heat. *Latent heat* Q_l is heat that changes liquid to vapor or vapor to liquid without a change in temperature or pressure of the moisture. Latent heat is also called the heat of vaporization or condensation. When water is vaporized, it absorbs heat which becomes latent heat. When the vapor condenses, latent heat is released, usually becoming sensible heat. *Total heat* Q_t is the sum of sensible and latent enthalpy change Δh [kJ/kg_{da}] multiplied by the mass of the dry air m_{da} [kg_{da}] (Figure 39.16-c). Total heat, or enthalpy, is usually measured from zero degrees Celsius for air.

To define the state of the air-vapor mixture, three independent properties are requested; one of these is the total pressure p . Once fixed, for instance $p = 101.325$ kPa (sea level)^{39.9}, two properties are sufficient to obtain the others. Then the thermodynamic state, i.e. the position on the chart, is defined by specifying the values of any two parameters on an intersecting scale, e.g. dry-bulb and wet-bulb temperature, or dry-bulb temperature and relative humidity. When lines are drawn on the chart indicating changes in psychrometric conditions, they are called process lines. The protractor is used to establish the direction, i.e. the slope, of a *process (or condition) line* on the psychrometric chart. The mechanics of constructing this line are discussed in more detail in *Examples 39.11* and *39.12*.

39.9 - A psychrometric chart will be exact at only one barometric pressure because basic equations used to prepare such charts depend on atmospheric pressure; in the chart, the barometric pressure at sea level (101.325 kPa) is considered. In case of elevations, significantly different from the sea level (e.g.: 1500 m), a new chart shall be considered.

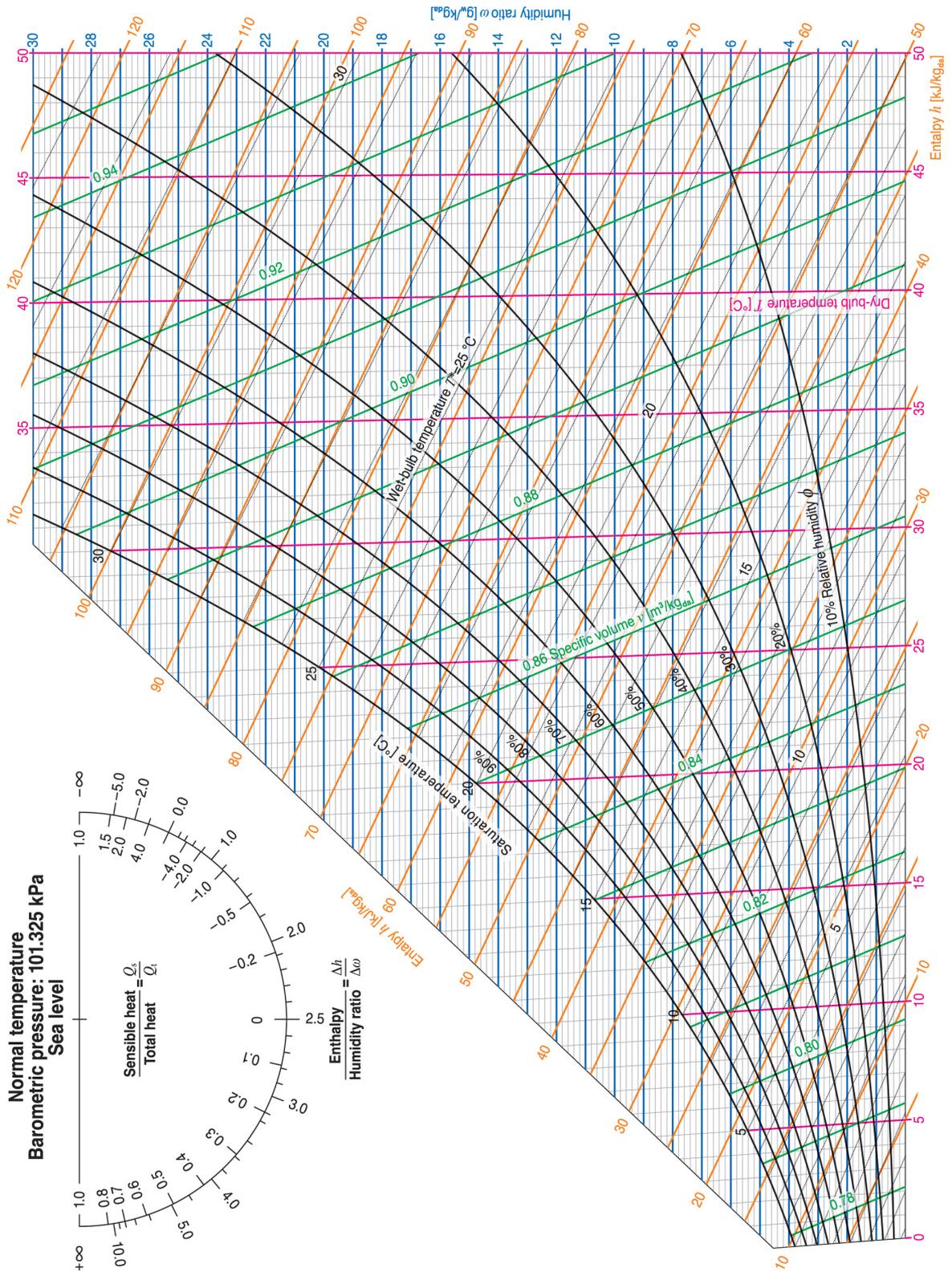


Fig. 39.16-b- Carrier's psychrometric chart. Here, the humidity ratio has been denoted by the symbol ϕ instead of x in accordance with international literature.

The psychrometric chart can be used to solve numerous process problems with moist air. Its use is best explained through illustrative examples. In each of the following examples, the process takes place at a constant total pressure of 101.325 kPa (101,325 Pa).

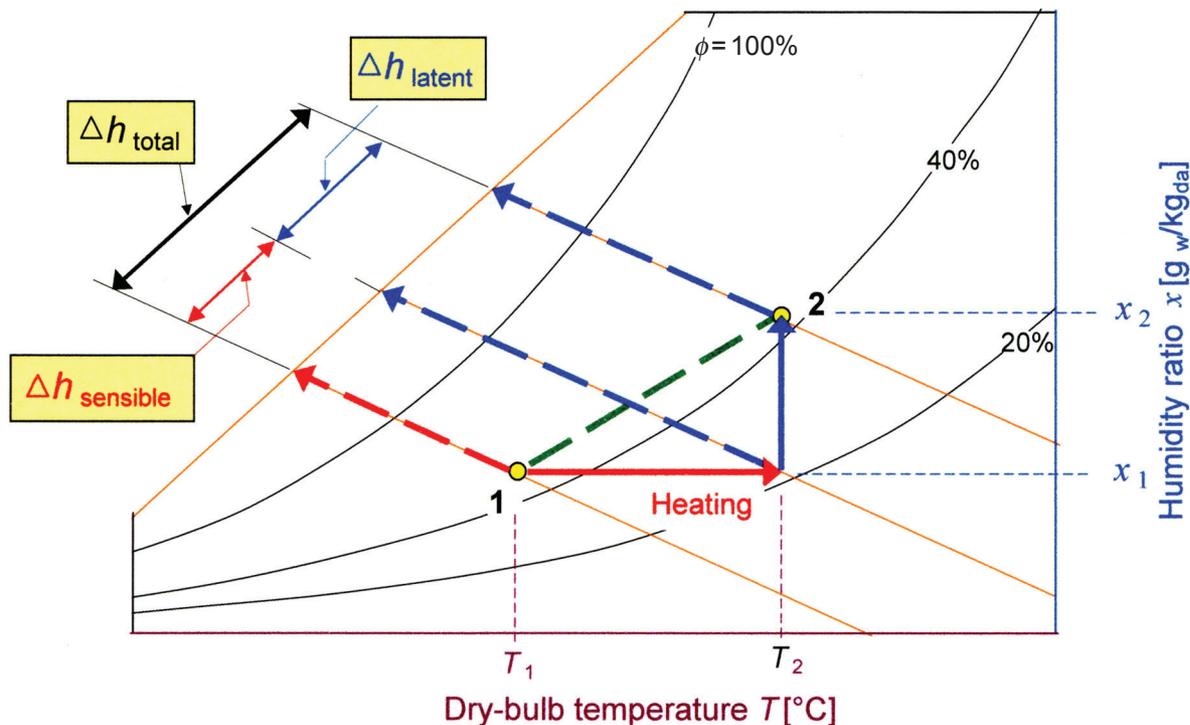


Fig. 39.16-c - Sensible, latent and total heat according to the process line 1-2 on psych chart. The thermal load \dot{Q} is the product of the dry air mass flow rate \dot{m}_{da} for the related enthalpy change Δh .

Example 39.5 Use of psych chart to determine moist air properties

The initial conditions of the moist air in a 150 m³ room are dry-bulb temperature $T_1 = 35$ °C and relative humidity $\phi_1 = 68\%$. Then the air is cooled in constant pressure process down to $T_2 = 20$ °C. Determine the air's:

- initial humidity ratio and dew-point temperature;
- final humidity ratio;
- condensed water vapor mass m_{w2} and dry air mass m_{da} ratio;
- heat ${}_1q_2$ [kJ per kg of dry air] to be subtracted from state 1 to state 2.

SOLUTION

- Locate point 1 where the 35°C purple vertical line intersects the curved 68% humidity black line (*Figure 39.17*); for this, interpolate between relative humidity curves of $\phi = 70\%$ and $\phi = 60\%$. Follow the horizontal blue line to the right until it intersects the humidity ratio scale. Read:

$$x_1 = 24.8 \text{ g}_w/\text{kg}_{da} = 0.0248 \text{ kg}_w/\text{kg}_{da}$$

The dew point must lie on a constant humidity ratio line because the water vapor, from the superheated region (point A, *Figure 39.13*) is not condensing and maintains its original mass; at the same time, it has to lie on the saturated vapor line (point B, *Figure 39.13*) to comply with the dew-point definition: the locus of these points is the 100% humidity curve, where $p_w = p_{ws}$ and $\phi = p_w/p_{ws} = 1 = 100\%$. Thus, follow the horizontal blue line to the left until it intersects the 100% humidity curve (point 1'), and read:

$$T_{d1} = 28.4 \text{ }^\circ\text{C} \quad \blacktriangleleft$$

- b) Follow the 100% relative humidity curve down to $T_2 = 20 \text{ }^\circ\text{C}$ and read, on the horizontal blue line, the humidity ratio:

$$x_2 = 0.015 \text{ kg}_w/\text{kg}_{da} \quad \blacktriangleleft$$

Please note that on the $\phi = 100\%$ relative humidity curve, the dew-point temperature is the same as the dry-bulb and wet-bulb temperature, because, on the saturated vapor line, the three temperatures shall coincide (*Paragraph 39.9*).

- c) The ratio between the condensed vapor mass m_{w2} and dry air mass m_{da} equals the difference of humidity ratio values between state 1 and 2:

$$\frac{m_{w2}}{m_{da}} = x_1 - x_2 = (0.0248 - 0.015) \frac{\text{kg}_w}{\text{kg}_{da}} = 0.0098 \frac{\text{kg}_w}{\text{kg}_{da}} = 9.8 \frac{\text{g}_w}{\text{kg}_{da}} \quad \blacktriangleleft$$

- d) The conservation of energy allows to calculate the heat ${}_1q_2$ needed to cool the mixture from 1 to 2; ${}_1q_2$ will result negative because it leaves the system. By assuming that the condensed water is cooled to the final temperature $T_2 = 20 \text{ }^\circ\text{C}$ before leaving the system, its enthalpy h_{w2} can be obtained by reading h_f on *Table A.3.1* or by multiplying the water specific $c_w = 4.186 \text{ kJ}/(\text{kg}\cdot^\circ\text{C})$ for T_2 . Being ${}_1q_2$ equal to the enthalpy difference, the enthalpy of the condensed water is very small ($0.82 \text{ kJ}/\text{kg}_{da}$) with respect to enthalpies of the air-vapor mixture can be read on the diagonal lines ($h_1 = 91 \text{ kJ}/\text{kg}_{da}$ and $h_2 = 58 \text{ kJ}/\text{kg}_{da}$).

$$\text{Table A.3.1: } h_{w2} = h_f = 83.96 \text{ kJ}/\text{kg}_w \quad \text{or} \quad h_{w2} = c_w T_2 \approx 4.186 \frac{\text{kJ}}{\text{kg}_w \cdot ^\circ\text{C}} \times 20 \text{ }^\circ\text{C} = 83.7 \frac{\text{kJ}}{\text{kg}_w}$$

$$(x_1 - x_2)h_{w2} = 0.0098 \frac{\text{kg}_w}{\text{kg}_{da}} \times 83.96 \frac{\text{kJ}}{\text{kg}_w} = 0.82 \frac{\text{kJ}}{\text{kg}_{da}}$$

$${}_1q_2 + h_1 = h_2 + (x_1 - x_2)h_{w2} \quad \Rightarrow \quad {}_1q_2 = h_2 - h_1 + (x_1 - x_2)h_{w2} \approx h_2 - h_1$$

$${}_1q_2 \approx h_2 - h_1 = (58 - 91) \frac{\text{kJ}}{\text{kg}_{da}} = -33 \frac{\text{kJ}}{\text{kg}_{da}} \quad \blacktriangleleft$$

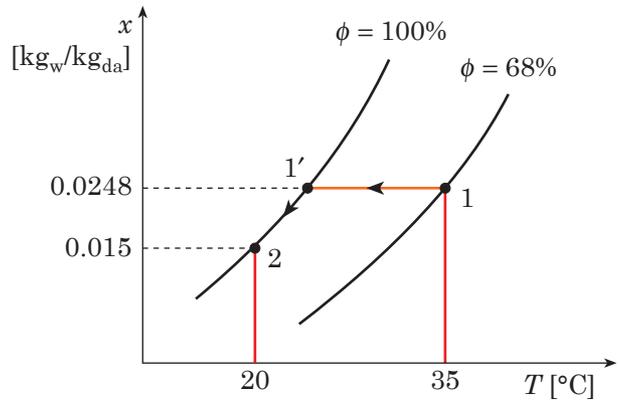


Fig. 39.17 - Example 39.5 solved by means of the psychrometric chart.

39.12 Air conditioning

39.12.1 Conservation equations of the moist air

Writing the material balance and steady-flow energy equations of a moist air system, the following quantities are considered:

- mass flow rate of dry air [$\text{kg}_{\text{da}}/\text{s}$] entering $\dot{m}_{\text{da,in}}$ and leaving $\dot{m}_{\text{da,out}}$ the system;
- mass flow rate [kg_w/s] of vapor \dot{m}_w or liquid water;
- rate of heat transfer \dot{Q} and rate of energy transfer as work \dot{W} [W].

The mass flow rate of dry air remains constant during air-conditioning, while it is the moist air that changes in the process. Thus, the material balance in terms of only dry air, let us maintain constant mass flow rate during humidification ($+\dot{m}_w$) or dehumidification ($-\dot{m}_w$). In the mass flow rate as water, the dry air mass flow rate \dot{m}_{da} [$\text{kg}_{\text{da}}/\text{s}$] is multiplied (39-10) for humidity ratio x [$\text{kg}_w/\text{kg}_{\text{da}}$] to obtain the water mass flow rate $\dot{m}_{\text{da}}x$ [kg_w/s].

By multiplying \dot{m}_{da} [$\text{kg}_{\text{da}}/\text{s}$] for enthalpy h , measured as kilojoule per kilogram of dry air [$\text{kJ}/\text{kg}_{\text{da}}$] a power is obtained [$\text{kJ}/\text{s} = \text{W}$]. At the same time, by multiplying \dot{m}_w [kg_w/s] for its enthalpy h_w [kJ/kg_w], a power is obtained, that usually is very low, mainly because (Example 39.5) small is the water mass flow rate with respect to dry air mass flow rate. In the power equation, corresponding to the energy conservation, the only rate of heat transfer \dot{Q} (>0 when entering and <0 when leaving the system) is taken into account, while the rate of energy transfer as work \dot{W} is not considered because negligible.

$$\left. \begin{aligned}
 \dot{m}_{\text{da,in}} &= \dot{m}_{\text{da,out}} && \ll \text{dry air balance} \gg \\
 \dot{m}_{\text{da,in}}x_{\text{in}} \pm \dot{m}_w &= \dot{m}_{\text{da,out}}x_{\text{out}} && \ll \text{water balance} \gg \\
 \dot{m}_{\text{da,in}}h_{\text{in}} \pm \dot{m}_wh_w + \dot{Q} &= \dot{m}_{\text{da,out}}h_{\text{out}} && \ll \text{energy conservation as power} \gg
 \end{aligned} \right\} \quad \mathbf{39-18}$$

The first equation should be reduced to $\dot{m}_{\text{da,in}} = \dot{m}_{\text{da,out}} = \dot{m}_{\text{da}}$, because usually there is only one mass flow rate entering and leaving the system. Exception is made by the adiabatic mixing of two moist airstreams of Sec. 39.12.4.

39.12.2 Sensible heating and cooling

Sensible heating or cooling is a process without subtraction or addition of water ($\dot{m}_w = 0$); it is the same as that occurring in a surface heat exchanger where the heating system can be a hot water coil or an electric heater and the refrigerating system can be a cool water coil or the evaporator of a refrigerator. The steady-flow energy and material balance equations (39-18) become:

$$\left. \begin{aligned} \dot{m}_{da,in} x_{in} \pm \dot{m}_w &= \dot{m}_{da,out} x_{out} \Rightarrow \dot{m}_{da} x_1 \pm 0 = \dot{m}_{da} x_2 \Rightarrow x_1 = x_2 && \ll \text{water} \gg \\ \dot{m}_{da,in} h_{in} \pm \dot{m}_w h_w + \dot{Q} &= \dot{m}_{da,out} h_{out} \Rightarrow \dot{m}_{da} h_1 \pm 0 + \dot{Q}_s = \dot{m}_{da} h_2 \Rightarrow \dot{Q}_s = \dot{m}_{da} (h_2 - h_1) && \ll \text{power} \gg \end{aligned} \right\} \text{39-19}$$

Adding heat alone to or withdrawing heat alone from moist air is represented by a horizontal line on the psych chart, because the humidity ratio x remains unchanged; at the same time the relative humidity ϕ is reduced when heating (Figure 39.18) or is increased when cooling (Figure 39.19). Thus, in the winter, when the outdoor air has to be heated, it is necessary to humidify it; or, in the summer, the temperature has not to be reduced below the dew-point temperature, otherwise part of vapor condenses and cooling should be associated to dehumidification.

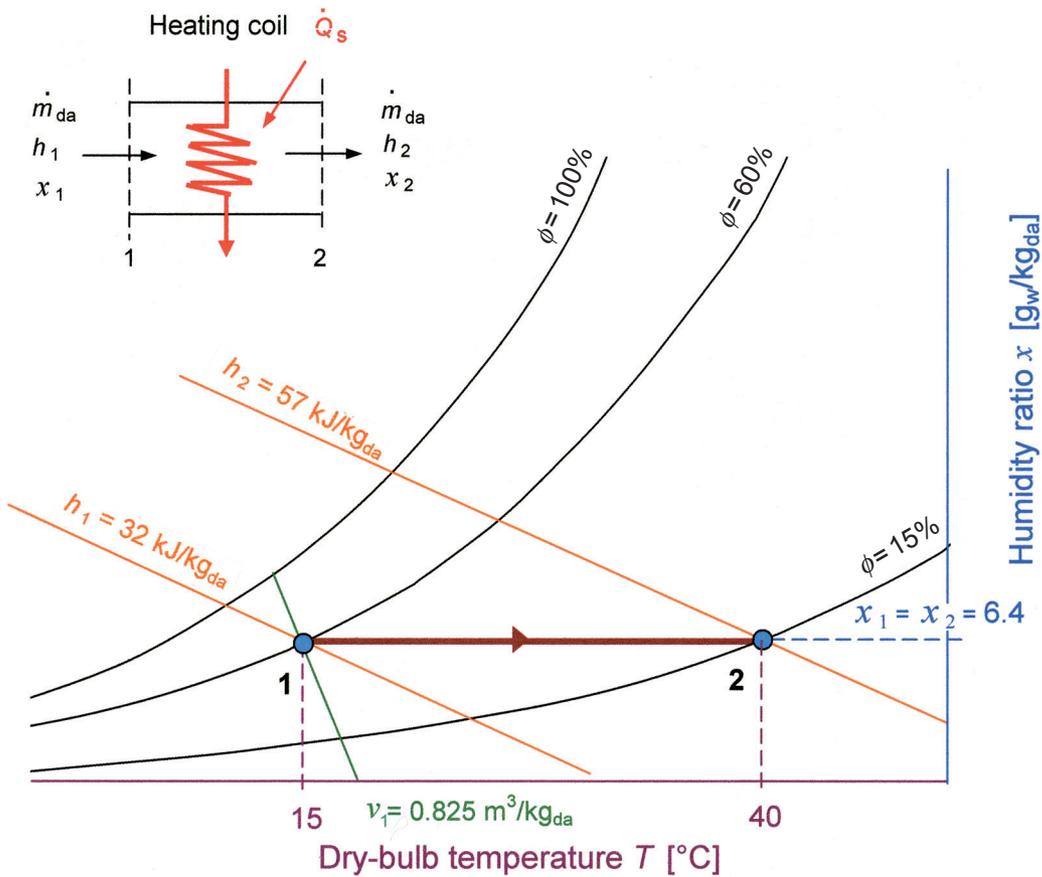


Fig. 39.18 - Heating without humidification on psychrometric chart: the relative humidity ϕ decreases.

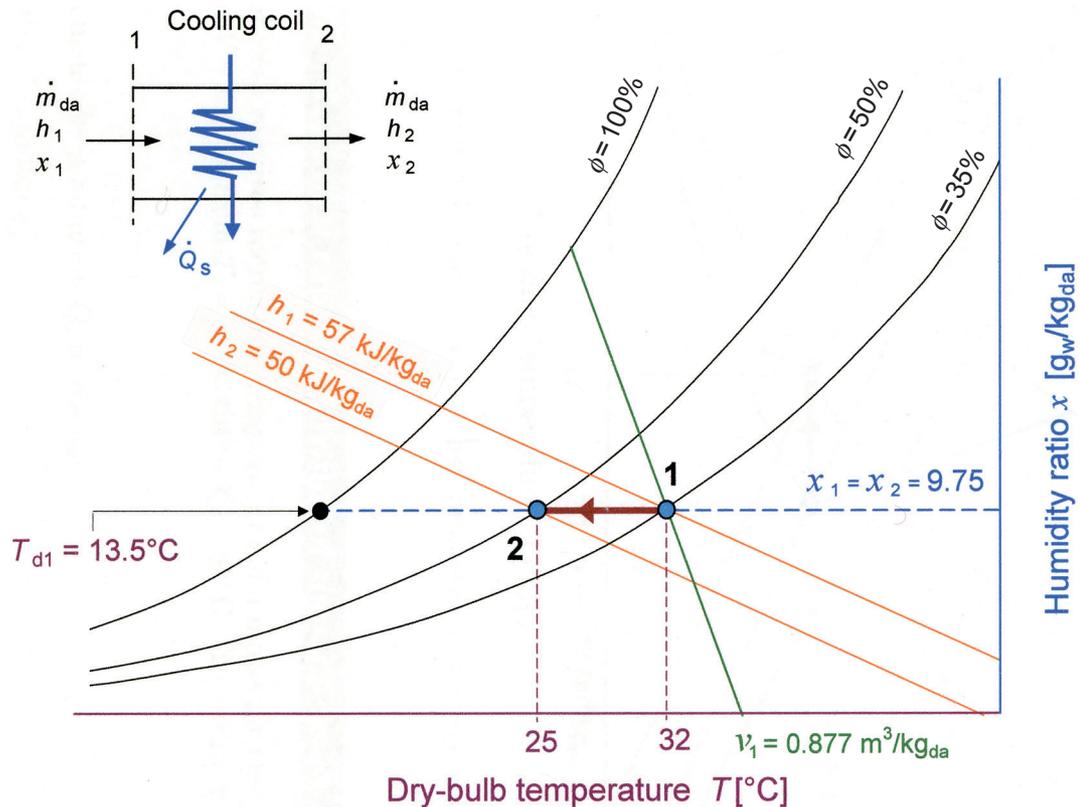


Fig. 39.19 - Cooling without dehumidification on psychrometric chart: the relative humidity ϕ increases.

Example 39.6 Required rate of addition of sensible heat

An industrial process requires increasing temperature of a moist air stream, with a volumetric flow rate $\dot{V} = 1 \text{ m}^3/\text{s}$, from $T_1 = 15 \text{ }^\circ\text{C}$ till to $T_2 = 40 \text{ }^\circ\text{C}$. The initial relative humidity is $\phi_1 = 60\%$.

Determine:

- the rate of sensible heat addition \dot{Q}_s to the air;
- the final relative humidity ϕ_2 .

SOLUTION

- Locate the point 1 (*Figure 39.18*) where the $T_1 = 15 \text{ }^\circ\text{C}$ vertical purple line intersects the curved $\phi_1 = 60\%$ humidity black line. Determine the other properties at point 1:
 - follow the orange diagonal line to the left until it intersects the enthalpy scale – read $h_1 = 32 \text{ kJ/kg}_{\text{da}}$;
 - interpolate between green diagonal lines of specific volume – read $v_1 = 0.825 \text{ m}^3/\text{kg}_{\text{da}}$;
 - follow the blue horizontal line to the right until it intersects the humidity ratio scale – read $x_1 = x_2 = 6.4 \text{ g}_w/\text{kg}_{\text{da}}$.

The dry air mass flow rate \dot{m}_{da} is:

$$\dot{m}_{\text{da}} = \frac{\dot{V}}{\nu_1} = \frac{1 \text{ m}^3/\text{s}}{0.825 \text{ m}^3/\text{kg}_{\text{da}}} = 1.21 \text{ kg/s}$$

From the point 1, follow the blue horizontal line to the right until it intersects the vertical purple line corresponding to $T_2 = 40^\circ\text{C}$. This is the point 2 (*Figure 39.18*). Follow the orange diagonal line to the left until it intersects the enthalpy scale – read $h_2 = 57 \text{ kJ/kg}_{\text{da}}$. The rate of sensible heat addition to the moist air is **(39-19)**:

$$\dot{Q}_s = \dot{m}_{\text{da}}(h_2 - h_1) = 1.21 \frac{\text{kg}_{\text{da}}}{\text{s}} \times (57 - 32) \frac{\text{kJ}}{\text{kg}_{\text{da}}} = 30.25 \text{ kW} \quad \blacktriangleleft$$

b) Interpolate between the black curved lines of relative humidity – read:

$$\phi_2 = 15\% \quad \blacktriangleleft$$

Example 39.7 Required rate of withdrawal of sensible heat

The temperatures of moist air in a building of 3600 m^3 are $T_1 = 32^\circ\text{C}$ (dry-bulb) and $T_1^* = 20^\circ\text{C}$ (wet-bulb). The air should be cooled down to $T_2 = 25^\circ\text{C}$, providing one air change per hour.

Determine:

- the initial conditions in terms of humidity ratio x_1 , relative humidity ϕ_1 , enthalpy h_1 , specific volume ν_1 and dew-point temperature T_{d1} ;
- the final conditions in terms of humidity ratio x_2 , relative humidity ϕ_2 , enthalpy h_2 and dew-point temperature T_{d2} ;
- the rate of sensible heat \dot{Q}_s to be withdrawn from the air.

SOLUTION

- On psychrometric chart (see *Paragraph 39.11*), determine the state 1 as intersection of purple vertical line $T_1 = 32^\circ\text{C}$ and the black diagonal dotted line $T_1^* = 20^\circ\text{C}$. Read the following values (*Figure 39.19*):

$$x_1 = 9.75 \text{ g}_w/\text{kg}_{\text{da}} \quad \phi_1 = 33\% \quad h_1 = 57 \text{ kJ/kg}_{\text{da}} \quad \nu_1 = 0.877 \text{ m}^3/\text{kg} \quad T_{\text{d1}} = 13.5^\circ\text{C} \quad \blacktriangleleft$$

where, for dew point temperature, follow the horizontal line to the left until it intersects the saturation temperature curve ($\phi = 100\%$), and then read the dew-point temperature by interpolating the purple vertical lines of dry-bulb temperature.

- Follow the blue horizontal line, $x_2 = x_1 = 9.75 \text{ g}_w/\text{kg}_{\text{da}}$, until it intersects the purple vertical line, $T_2 = 25^\circ\text{C}$. Read (*Figure 39.19*):

$$\phi_2 = 50\% \quad h_2 = 50 \text{ kJ/kg}_{\text{da}} \quad T_{\text{d2}} = T_{\text{d1}} = 13.5^\circ\text{C} \quad \blacktriangleleft$$

- An air change per hour (1 h = 3600 s) means a volume flow rate $\dot{V} = 3600 \text{ m}^3/3600 \text{ s} = 1 \text{ m}^3/\text{s}$. From the steady-flow energy equation, \dot{Q}_s is **(39-19)** the product of mass flow rate of dry

air, \dot{m}_{da} , for enthalpy difference, $\Delta h = (h_2 - h_1)$. The rate of withdrawal of sensible heat is negative because it leaves the system.

$$\dot{m}_{da} = \frac{\dot{V}}{\nu_1} = \frac{1 \text{ m}^3/\text{s}}{0.877 \text{ m}^3/\text{kg}_{da}} = 1.14 \text{ kg}_{da}/\text{s}$$

$$\dot{Q}_s = \dot{m}_{da} (h_1 - h_2) = 1.14 \frac{\text{kg}_{da}}{\text{s}} \times (50 - 57) \frac{\text{kJ}}{\text{kg}_{da}} = -8 \text{ kW}$$

39.12.3 Moist air cooling and dehumidification

Moisture condensation occurs when moist air is cooled to a temperature below its initial dew point. *Figure 39.20* shows a schematic cooling coil where moist air is assumed to be uniformly processed. On the psychrometric chart, the moist air is cooled from the initial state 1 to the point 2' with relative humidity $\phi = 100\%$ and dew point temperature $T_{d2'}$; then, the cooling goes on meeting state 2 on $\phi = 100\%$ curve. Although water can be withdrawn at various temperatures ranging from the initial dew point to the final saturation temperature (broken line curve 1-2), it is assumed that condensed water is cooled to the final air temperature T_2 before it drains from the system. The steady-flow energy and material balance equations 39-18 become:

$$\left. \begin{aligned} \dot{m}_{da} x_1 - \dot{m}_w &= \dot{m}_{da} x_2 \Rightarrow \dot{m}_w = \dot{m}_{da} (x_1 - x_2) && \ll \text{water} \gg \\ \dot{m}_{da} h_1 - \dot{m}_w h_{w2} + \dot{Q} &= \dot{m}_{da} h_2 \Rightarrow h_2 = h_1 - (x_1 - x_2) h_{w2} + \frac{\dot{Q}}{\dot{m}_{da}} && \ll \text{power} \gg \end{aligned} \right\} \quad \mathbf{39-20}$$

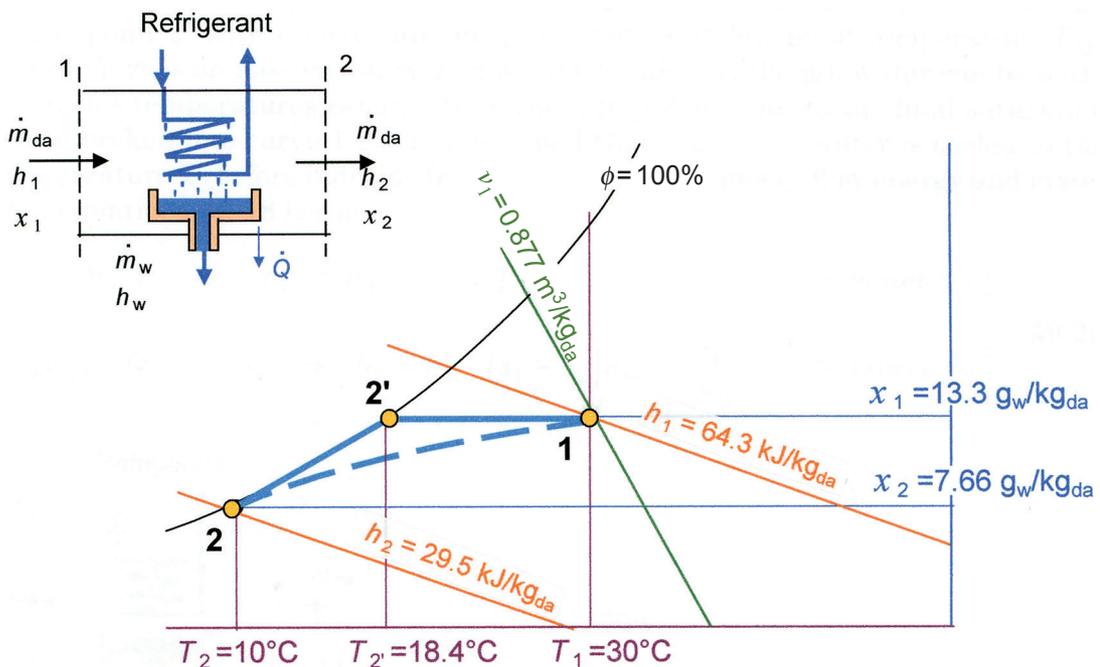


Fig. 39.20 - Moist air cooling and dehumidification on psychrometrics chart.

In the cooling process with dehumidification, only part of the air comes into contact with the cool surface and process 1-2 is effected (*Figure 39.21*). The part of air that passes through the cooler unchanged divided by the total air is called bypass factor (*BPF*). The air temperature T_2 equals *BPF* multiplied by T_1 plus $(1 - BPF)$ multiplied by T_S , that is the average surface temperature of the cooler:

$$T_2 = BPF \cdot T_1 + (1 - BPF) \cdot T_S \Rightarrow BPF \cdot T_1 + T_S - BPF \cdot T_S - T_2 = 0 \Rightarrow BPF = \frac{T_2 - T_S}{T_1 - T_S}$$

Instead of the temperature $(T_2 - T_S)/(T_1 - T_S)$, other properties such as humidity ratio x or enthalpy h should be considered. About the power conservation equation (39-20), where the term $(x_1 - x_2)h_{w2}$ related to the condensed water is negligible, the rate of heat withdrawn from the moist air \dot{Q}_t (*Figure 39.21*) is used part to cool the air via the sensible heat rate, $\dot{Q}_s = \dot{m}_{da}(h_2 - h_3)$, and part to humidify the air via the latent heat rate, $\dot{Q}_1 = \dot{m}_{da}(h_3 - h_1)$.

$$BPF = \frac{T_2 - T_S}{T_1 - T_S} \quad \dot{Q}_t \cong \dot{m}_{da}(h_2 - h_1) = \dot{Q}_s + \dot{Q}_1 = \dot{m}_{da}(h_2 - h_3) + \dot{m}_{da}(h_3 - h_1) \quad 39-21$$

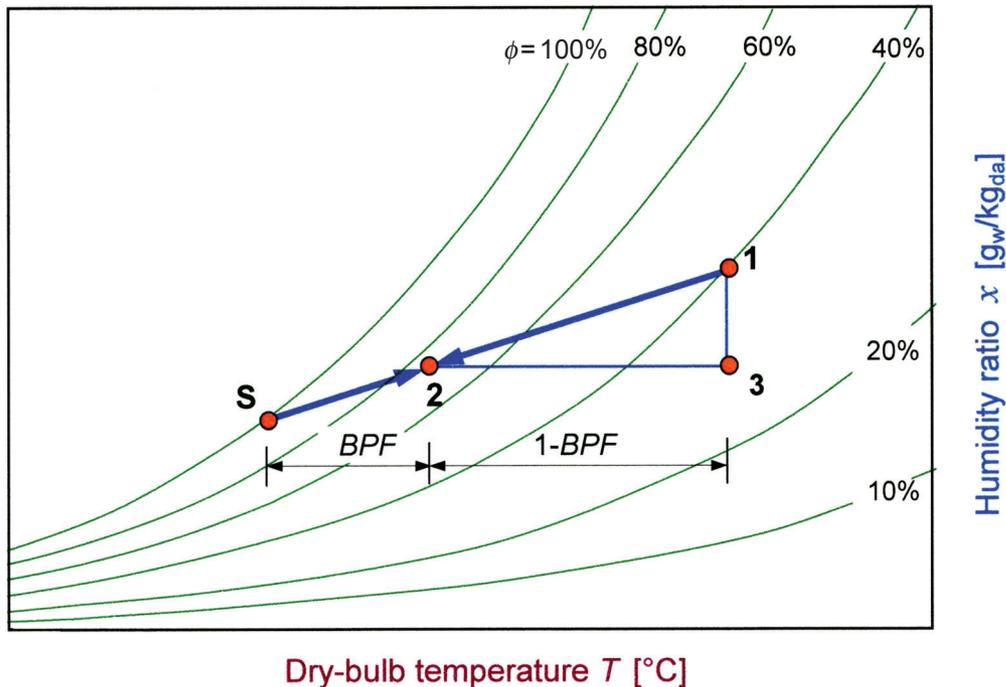


Fig. 39.21 - Cooling with dehumidification and bypass factor (*BPF*).

Example 39.8 Moist air cooling and dehumidification

Moist air at $T_1 = 30\text{ }^\circ\text{C}$ dry-bulb temperature and $\phi_1 = 50\%$ relative humidity enters a cooling coil with $\dot{V} = 5\text{ m}^3/\text{s}$ and is processed to a final saturation condition at $T_2 = 10\text{ }^\circ\text{C}$.

Find the power \dot{Q} of refrigeration required.

SOLUTION

Point 1 is found as intersection of temperature $T_1 = 30\text{ }^\circ\text{C}$ with $\phi_1 = 50\%$; from the psychrometric chart, other properties of state 1 are: enthalpy $h_1 = 64.3\text{ kJ/kg}_{\text{da}}$, humidity ratio $x_1 = 13.3\text{ g}_w/\text{kg}_{\text{da}}$ and specific volume $\nu_1 = 0.877\text{ m}^3/\text{kg}_{\text{da}}$. State 2 is located on the saturation curve ($\phi_2 = 100\%$) at $T_2 = 10\text{ }^\circ\text{C}$; in the psychrometric chart, read $h_2 = 29.5\text{ kJ/kg}_{\text{da}}$ and $x_2 = 7.66\text{ g}_w/\text{kg}_{\text{da}}$. Assuming that condensed water is cooled to the final air temperature T_2 before it drains from the system, its enthalpy h_{w2} is obtained from *Table A.3.1*: $h_{w2} = h_f = 42.01\text{ kJ/kg}_w$. Once known the mass flow of dry air \dot{m}_{da} from \dot{V}/ν_1 , the refrigeration power \dot{Q} can be obtained; it is negative because referring to heat leaving the system.

$$\dot{m}_{\text{da}} = \frac{\dot{V}}{\nu_1} = \frac{5\text{ m}^3/\text{s}}{0.877\text{ m}^3/\text{kg}_{\text{da}}} = 5.7\text{ kg}_{\text{da}}/\text{s}$$

$$\begin{aligned} \mathbf{39-20:} \quad h_2 &= h_1 - (x_1 - x_2)h_{w2} + \frac{\dot{Q}}{\dot{m}_{\text{da}}} \Rightarrow \dot{Q} = \dot{m}_{\text{da}} \left[(h_2 - h_1) + (x_1 - x_2)h_{w2} \right] = \\ &= 5.7 \frac{\text{kg}_{\text{da}}}{\text{s}} \left[(29.5 - 64.3) \frac{\text{kJ}}{\text{kg}_{\text{da}}} + (0.0133 - 0.00766) \frac{\text{kg}_w}{\text{kg}_{\text{da}}} \times 42.01 \frac{\text{kJ}}{\text{kg}_w} \right] = \\ &= 5.7 \frac{\text{kg}_{\text{da}}}{\text{s}} \left[-34.8 \frac{\text{kJ}}{\text{kg}_{\text{da}}} + 0.24 \frac{\text{kJ}}{\text{kg}_{\text{da}}} \right] = -197\text{ kW} \end{aligned}$$

Once again (*Example 39.5*) it should be noted how negligible is the liquid enthalpy ($0.24\text{ kJ/kg}_{\text{da}}$).

Example 39.9 Dehumidification with bypass

A mass rate of dry air $\dot{m}_{\text{da}} = 0.1\text{ kg}_{\text{da}}/\text{s}$ flows through a dehumidifier with average surface temperature $T_s = 12\text{ }^\circ\text{C}$ and bypass factor $BPF = 0.1$. The outdoor air is at temperature $T_1 = 24\text{ }^\circ\text{C}$ and relative humidity $\phi_1 = 70\%$. By using the psychrometric chart, find the properties of moist air leaving the dehumidifier (state 2 in *Figure 39.21*), the total heat power \dot{Q}_t and the mass flow rate of the condensed water \dot{m}_w .

SOLUTION

Table, at the bottom, shows the properties of states 1 and S. T_2 , x_2 , h_2 and \dot{Q}_t are calculated by means of **39-21**, while the relative humidity ϕ_2 is read on psych chart. The condensed water flow rate \dot{m}_w is calculated by means of **39-20**.

$$BPF = \frac{T_2 - T_S}{T_1 - T_S} \Rightarrow T_2 = BPF \cdot (T_1 - T_S) + T_S = 0.1 \times (24 - 12)^\circ\text{C} + 12^\circ\text{C} = 13.2^\circ\text{C}$$

$$BPF = \frac{x_2 - x_S}{x_1 - x_S} \Rightarrow x_2 = BPF \cdot (x_1 - x_S) + x_S = 0.1 \times (13.0 - 8.7) \frac{\text{g}_w}{\text{kg}_{\text{da}}} + 8.7 \frac{\text{g}_w}{\text{kg}_{\text{da}}} = 9.1 \frac{\text{g}_w}{\text{kg}_{\text{da}}}$$

$$BPF = \frac{h_2 - h_S}{h_1 - h_S} \Rightarrow h_2 = BPF \cdot (h_1 - h_S) + h_S = 0.1 \times (57.5 - 34.0) \frac{\text{kJ}}{\text{kg}_{\text{da}}} + 34.0 \frac{\text{kJ}}{\text{kg}_{\text{da}}} = 36.4 \frac{\text{kJ}}{\text{kg}_{\text{da}}}$$

$$\dot{Q}_t \cong \dot{m}_{\text{da}} (h_2 - h_1) = 0.1 \text{ kg}_{\text{da}}/\text{s} \times (36.4 - 57.5) \text{ kJ/kg}_{\text{da}} = -2.1 \text{ kW} \quad \blacktriangleleft$$

$$\dot{m}_w = \dot{m}_{\text{da}} (x_1 - x_2) = 0.1 \text{ kg}_{\text{da}}/\text{s} \times (0.0130 - 0.0091) \text{ kg}_w/\text{kg}_{\text{da}} = 0.39 \text{ g}_w/\text{s} \quad \blacktriangleleft$$

State ^(*)	T [°C]	φ [%]	x [g _w /kg _{da}]	h [kJ/kg _{da}]
1	24	70	13.2	57.5
S	12	100	8.7	34.0
2	13.2	95	9.1	36.4

(*) Black values are known data; red values are calculated or read on psychrometric chart.

39.12.4 Adiabatic mixing of two moist airstreams

A common process in air-conditioning systems is the adiabatic mixing of two moist airstreams. The mass flow rate $\dot{m}_{\text{da},\text{in}}$ entering the system is composed by two streams $\dot{m}_{\text{da}1}$ and $\dot{m}_{\text{da}2}$ mixing adiabatically ($\dot{Q} = 0$), while the mass flow rate leaving the system is $\dot{m}_{\text{da},\text{out}} = \dot{m}_{\text{da}3}$, similar to the mode of operation of a direct-contact heat exchanger (Figure 39.22). Adiabatic mixing is governed by three equations derived from 39-18. These equations state that the humidity ratio x_3 and enthalpy h_3 leaving the system are the weighted mean of total mass flow rate \dot{m}_{da} .

$$\left. \begin{aligned} \dot{m}_{\text{da}1} + \dot{m}_{\text{da}2} = \dot{m}_{\text{da}3} &\Rightarrow \dot{m}_{\text{da}} = \dot{m}_{\text{da}3} = \dot{m}_{\text{da}1} + \dot{m}_{\text{da}2} && \ll \text{dry air} \gg \\ \dot{m}_{\text{da}1}x_1 + \dot{m}_{\text{da}2}x_2 = \dot{m}_{\text{da}3}x_3 &\Rightarrow \frac{x_2 - x_3}{x_3 - x_1} = \frac{\dot{m}_{\text{da}1}}{\dot{m}_{\text{da}2}} \Rightarrow x_3 = \frac{\dot{m}_{\text{da}1}x_1 + \dot{m}_{\text{da}2}x_2}{\dot{m}_{\text{da}}} && \ll \text{water} \gg \\ \dot{m}_{\text{da}1}h_1 + \dot{m}_{\text{da}2}h_2 = \dot{m}_{\text{da}3}h_3 &\Rightarrow \frac{h_2 - h_3}{h_3 - h_1} = \frac{\dot{m}_{\text{da}1}}{\dot{m}_{\text{da}2}} \Rightarrow h_3 = \frac{\dot{m}_{\text{da}1}h_1 + \dot{m}_{\text{da}2}h_2}{\dot{m}_{\text{da}}} && \ll \text{power} \gg \end{aligned} \right\} \text{39-22}$$

$$\dot{m}_{\text{da}1}h_1 + \dot{m}_{\text{da}2}h_2 = \dot{m}_{\text{da}3}h_3 \Rightarrow h_3 = \frac{\dot{m}_{\text{da}1}h_1 + \dot{m}_{\text{da}2}h_2}{\dot{m}_{\text{da}}}$$

$$\dot{m}_{\text{da}1}h_1 + \dot{m}_{\text{da}2}h_2 = \dot{m}_{\text{da}3}h_3 \Rightarrow \dot{m}_{\text{da}1}h_1 + \dot{m}_{\text{da}2}h_2 = (\dot{m}_{\text{da}1} + \dot{m}_{\text{da}2})h_3 \Rightarrow$$

$$\Rightarrow \dot{m}_{da2}(h_2 - h_3) = \dot{m}_{da1}(h_3 - h_1) \Rightarrow \frac{h_2 - h_3}{h_3 - h_1} = \frac{\dot{m}_{da1}}{\dot{m}_{da2}}$$

or

$$\dot{m}_{da1}h_1 + \dot{m}_{da2}h_2 = \dot{m}_{da3}h_3 \Rightarrow (\dot{m}_{da3} - \dot{m}_{da2})h_1 + \dot{m}_{da2}h_2 = \dot{m}_{da3}h_3 \Rightarrow$$

$$\Rightarrow \dot{m}_{da3}h_1 - \dot{m}_{da2}h_1 - \dot{m}_{da3}h_3 + \dot{m}_{da2}h_2 = 0 \Rightarrow \frac{h_1 - h_3}{h_1 - h_2} = \frac{\dot{m}_{da2}}{\dot{m}_{da3}}$$

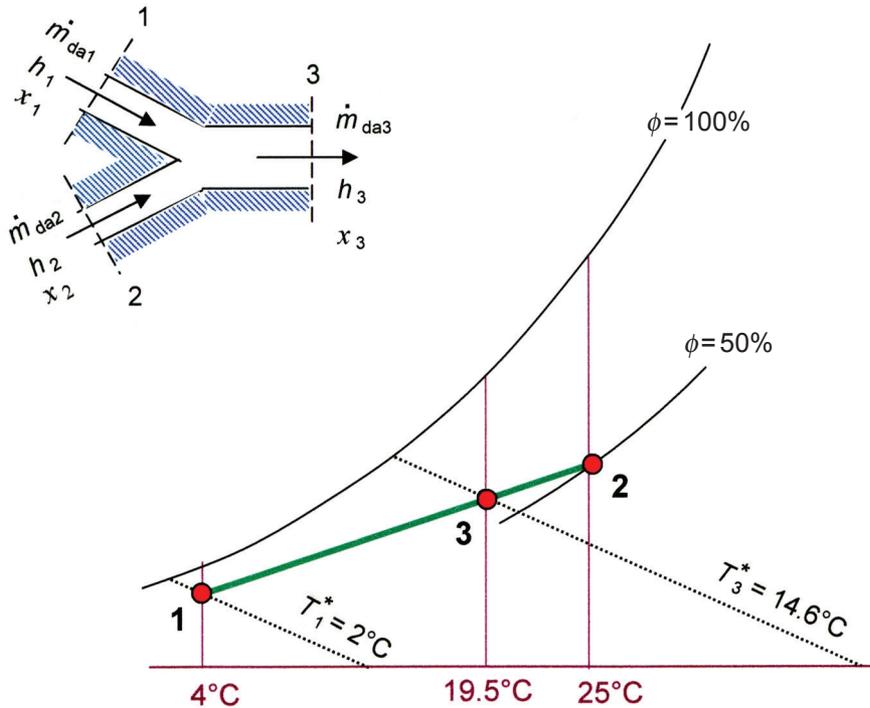


Fig. 39.22 - Adiabatic mixing of two moist airstreams. The mixture state (point 3) is on the line connecting states 1 and 2 of the two streams. The ratio between the lengths of line segments 1-3 and 1-2 is equal to the ratio of mass flow rate $\dot{m}_{da2}/\dot{m}_{da3}$, because the last equation 39-22 can be written $(h_1 - h_3)/(h_1 - h_2) = \dot{m}_{da2}/\dot{m}_{da3}$. If, for example, state 1 ($T_1 = 4^\circ\text{C}$, $T_1^* = 2^\circ\text{C}$ and $\dot{m}_{da1} = 2.535 \text{ kg}_{da}/\text{s}$) and state 2 ($T_2 = 25^\circ\text{C}$, $\phi_2 = 50\%$ and $\dot{m}_{da2} = 7.284 \text{ kg}_{da}/\text{s}$), are known, the mass flow rate is $\dot{m}_{da} = \dot{m}_{da3} = \dot{m}_{da1} + \dot{m}_{da2} = 9.819 \text{ kg}_{da}/\text{s}$. Being $\dot{m}_{da2}/\dot{m}_{da} = 0.742$, the length of line segment 1-3 is 0.742 times the length of entire line 1-2. Using a ruler, state 3 is located, and the values $T_3 = 19.5^\circ\text{C}$ and $T_3^* = 14.6^\circ\text{C}$ are found.

Example 39.10 Adiabatic mixing of two moist airstreams

A stream of $\dot{m}_{da1} = 0.78 \text{ kg}_{da}/\text{s}$ of outdoor air at $T_1 = 32^\circ\text{C}$ dry-bulb temperature and relative humidity $\phi_1 = 60\%$ is adiabatically mixed with $\dot{m}_{da2} = 1.63 \text{ kg}_{da}/\text{s}$ of recirculated air at $T_2 = 26^\circ\text{C}$ dry-bulb temperature and $\phi_2 = 50\%$ relative humidity. Find the dry-bulb temperature T_3 of the resulting mixture.

SOLUTION

Dry-bulb temperature and relative humidity of the states 1 and 2 of the two airstreams to be mixed and their mass flow rate are known. Other properties of the states 1 and 2 can be obtained via equations of *Paragraph 39.10* or by reading the psychrometric chart (*Figure 39.16-b*). With equations **39-22**, mass flow rate \dot{m}_3 , humidity ratio x_3 and enthalpy h_3 are calculated. Table at the bottom shows the results.

$$\mathbf{39-13:} \quad p_{ws} \cong 1000 \cdot e^{\left(\frac{16.6536 - \frac{4030.183}{T + 235}}{T + 235} \right)}$$

$$p_{ws1} \cong 1000 \cdot e^{\left(\frac{16.6536 - \frac{4030.183}{32^\circ\text{C} + 235}}{32^\circ\text{C} + 235} \right)} = 4755 \text{ Pa} \quad p_{ws2} \cong 1000 \cdot e^{\left(\frac{16.6536 - \frac{4030.183}{26^\circ\text{C} + 235}}{26^\circ\text{C} + 235} \right)} = 3361 \text{ Pa} \quad \blacktriangleleft$$

$$\mathbf{39-14:} \quad x = 0.622 \frac{\phi p_{ws}}{p - \phi p_{ws}}$$

$$x_1 = 0.622 \frac{0.60 \times 4755 \text{ Pa}}{101,325 \text{ Pa} - 0.60 \times 4755 \text{ Pa}} = 0.0180 \quad x_2 = 0.622 \frac{0.5 \times 3361 \text{ Pa}}{101,325 \text{ Pa} - 3361 \text{ Pa}} = 0.0105 \quad \blacktriangleleft$$

$$\mathbf{39-15:} \quad h = h_{da} + xh_g \cong 1.006 \cdot T + x(2501 + 1.86 \cdot T)$$

$$h_1 = 1.006 \frac{\text{kJ}}{\text{kg}_{da} \cdot ^\circ\text{C}} \times 32^\circ\text{C} + 0.0180 \frac{\text{kg}_w}{\text{kg}_{da}} \times \left(2501 \frac{\text{kJ}}{\text{kg}_w} + 1.86 \frac{\text{kJ}}{\text{kg}_{da} \cdot ^\circ\text{C}} \times 32^\circ\text{C} \right) = 78.3 \frac{\text{kJ}}{\text{kg}_{da}} \quad \blacktriangleleft$$

$$h_2 = 1.006 \frac{\text{kJ}}{\text{kg}_{da} \cdot ^\circ\text{C}} \times 26^\circ\text{C} + 0.0105 \frac{\text{kg}_w}{\text{kg}_{da}} \times \left(2501 \frac{\text{kJ}}{\text{kg}_{da}} + 1.86 \frac{\text{kJ}}{\text{kg}_{da} \cdot ^\circ\text{C}} \times 26^\circ\text{C} \right) = 52.9 \frac{\text{kJ}}{\text{kg}_{da}} \quad \blacktriangleleft$$

$$\mathbf{39-22:} \quad \left\{ \begin{array}{l} \dot{m}_{da} = \dot{m}_{da3} = \dot{m}_{da1} + \dot{m}_{da2} = 0.78 \text{ kg}_{da}/\text{s} + 1.63 \text{ kg}_{da}/\text{s} = 2.41 \text{ kg}_{da}/\text{s} \quad \blacktriangleleft \\ x_3 = \frac{\dot{m}_{da1}x_1 + \dot{m}_{da2}x_2}{\dot{m}_{da}} = \frac{0.78 \frac{\text{kg}_{da}}{\text{s}} \times 0.0180 \frac{\text{kg}_w}{\text{kg}_{da}} + 1.63 \frac{\text{kg}_{da}}{\text{s}} \times 0.0105 \frac{\text{kg}_w}{\text{kg}_{da}}}{2.41 \text{ kg}_{da}/\text{s}} = 0.013 \frac{\text{kg}_w}{\text{kg}_{da}} \quad \blacktriangleleft \\ h_3 = \frac{\dot{m}_{da1}h_1 + \dot{m}_{da2}h_2}{\dot{m}_{da}} = \frac{0.78 \frac{\text{kg}_{da}}{\text{s}} \times 78.3 \frac{\text{kJ}}{\text{kg}_{da}} + 1.63 \frac{\text{kg}_{da}}{\text{s}} \times 52.9 \frac{\text{kJ}}{\text{kg}_{da}}}{2.41 \text{ kg}_{da}/\text{s}} = 61.1 \frac{\text{kJ}}{\text{kg}_{da}} \quad \blacktriangleleft \end{array} \right.$$

$$\mathbf{39-15':} \quad T_3 = \frac{h_3 - 2501 \cdot x_3}{1.006 + 1.86 \cdot x_3} = \frac{61.1 \frac{\text{kJ}}{\text{kg}_{da}} - 2501 \frac{\text{kJ}}{\text{kg}_w} \times 0.013 \frac{\text{kg}_w}{\text{kg}_{da}}}{1.006 \frac{\text{kJ}}{\text{kg}_{da} \cdot ^\circ\text{C}} + 1.86 \frac{\text{kJ}}{\text{kg}_w \cdot ^\circ\text{C}} \times 0.013 \frac{\text{kg}_w}{\text{kg}_{da}}} = 27.7^\circ\text{C}$$

$$\mathbf{39-13:} \quad p_{ws3} \cong 1000 \cdot e^{\left(\frac{16.6536 - \frac{4030.183}{T_3 + 235}}{T_3 + 235} \right)} = 1000 \cdot e^{\left(\frac{16.6536 - \frac{4030.183}{28.1^\circ\text{C} + 235}}{28.1^\circ\text{C} + 235} \right)} = 3802 \text{ Pa}$$

$$\left. \begin{aligned} \dot{m}_{\text{da}}x_1 + \dot{m}_{\text{w}} &= \dot{m}_{\text{da}}x_2 \Rightarrow \dot{m}_{\text{w}} = \dot{m}_{\text{da}}(x_2 - x_1) && \ll \text{water} \gg \\ \dot{m}_{\text{da}}h_1 + \dot{m}_{\text{w}}h_{\text{w}} &= \dot{m}_{\text{da}}h_2 \Rightarrow \frac{h_2 - h_1}{x_2 - x_1} = \frac{\Delta h}{\Delta x} = h_{\text{w}} && \ll \text{power} \gg \end{aligned} \right\} \quad \mathbf{39-23}$$

$$\dot{m}_{\text{da}}h_1 + \dot{m}_{\text{w}}h_{\text{w}} = \dot{m}_{\text{da}}h_2 \Rightarrow \dot{m}_{\text{da}}h_1 + \dot{m}_{\text{da}}(x_2 - x_1)h_{\text{w}} = \dot{m}_{\text{da}}h_2 \Rightarrow \frac{h_2 - h_1}{x_2 - x_1} = \frac{\Delta h}{\Delta x} = h_{\text{w}}$$

According to the last equation, the final state point of the moist air lies (psych chart) on a straight line in the direction fixed by the specific enthalpy h_{w} of the injected water, drawn through the initial state point 1 of the moist air.

Example 39.11 Adiabatic mixing of water injected into moist air

Moist air at $T_1 = 20^\circ\text{C}$ dry-bulb and $T_1^* = 8^\circ\text{C}$ wet-bulb temperature is to be processed to a final dew-point temperature $T_{2\text{d}} = 13^\circ\text{C}$ by adiabatic injection of saturated steam at 110°C . The rate of dry airflow is $2 \text{ kg}_{\text{da}}/\text{s}$. Find the final dry-bulb temperature of the moist air T_2 and the rate of steam flow \dot{m}_{w} .

SOLUTION

On the psychrometric chart (*Figure 39.23*), the state 1 is established by intersecting the dry-bulb temperature $T_1 = 20^\circ\text{C}$ with the diagonal line of wet-bulb temperature $T_1^* = 8^\circ\text{C}$; at point 1, the humidity ratio is $x_1 = 1,8 \text{ g}_{\text{w}}/\text{kg}_{\text{da}}$. Locate the dew-point temperature $T_{2\text{d}} = 13^\circ\text{C}$ where the 13°C purple vertical line intersects the $\phi = 100\%$ relative humidity black curve. Read final humidity ratio $x_2 = 9,3 \text{ g}_{\text{w}}/\text{kg}_{\text{da}}$ on the blue horizontal line.

The enthalpy of saturated vapor at 110°C , is (*Table A.3.1*) $h_{\text{g}} = 2691,5 \text{ kJ}/\text{kg}_{\text{w}} = 2,691 \text{ kJ}/\text{g}_{\text{w}}$; the enthalpy has to be converted in $\text{kJ}/\text{g}_{\text{w}}$ because the humidity ratio scale on the psychrometric chart is $\text{g}_{\text{w}}/\text{kg}_{\text{da}}$. According to equation **39-23**, the *process* (or *condition*) green line on the psychrometric chart connecting states 1 and 2 must have a *direction* $\Delta h/\Delta x = h_{\text{w}} = 2,691 \text{ kJ}/\text{g}_{\text{w}}$. The process line can be drawn with the $\Delta h/\Delta x$ protractor. First, establish the reference line on the protractor by connecting the origin with the value $\Delta h/\Delta x = 2,691 \text{ kJ}/\text{g}_{\text{w}}$. Draw a second line parallel to the reference line and through the initial state point 1 of the moist air. This second green line is the process line. State 2 is established at the intersection of the process line with the blue horizontal line extended from the saturation curve at $T_{2\text{d}} = 13^\circ\text{C}$. Thus the dry-bulb temperature $T_2 = 21^\circ\text{C}$ is obtained, while the required steam flow \dot{m}_{w} is calculated with equation **39-23**.

$$T_2 = 21^\circ\text{C} \quad \dot{m}_{\text{w}} = \dot{m}_{\text{da}}(x_2 - x_1) = 2 \frac{\text{kg}_{\text{da}}}{\text{s}} \times (0,0093 - 0,0018) \frac{\text{kg}_{\text{w}}}{\text{kg}_{\text{da}}} = 0,015 \frac{\text{kg}_{\text{w}}}{\text{s}} \quad \blacktriangleleft$$

39.12.6 Space heat absorption and moisture gains

The general case of air conditioning required for a space is determined by:

- the quantity of moist air to be supplied;
- the supply air condition necessary to remove given amounts of energy and water from the space.

In a space, where incident rates of energy and moisture gains are involved as shown in *Figure 39.24*, two main quantities, introduced in *Paragraph 39.11*, are considered: *sensible heat load* \dot{Q}_s and *latent heat load* \dot{Q}_l . \dot{Q}_s denotes the net sum of all rates of heat gain in the space, arising from transfers through boundaries and from sources within the space; the heat gain involves energy addition alone and does not include energy contributions from water (or water vapor) addition. \dot{Q}_l depends on the quantity \dot{m}_w related to the net sum of all rates of moisture gain on the space arising from transfers through boundaries and from sources within the space; the mass flow rate of water vapor \dot{m}_w adds an amount of energy equal to its specific enthalpy h_w : $\dot{Q}_l = \dot{m}_w h_w$. The sum of the rate of sensible heat load \dot{Q}_s and the latent heat load \dot{Q}_l is the *total heat load* \dot{Q}_t for the space.

$$\dot{Q}_l = \dot{m}_w h_w \quad \mathbf{39-24} \qquad \dot{Q}_t = \dot{Q}_s + \dot{Q}_l \quad \mathbf{39-25}$$

Assuming steady-state conditions, governing equations of the system in *Figure 39.24* are (see equations **39-18**):

$$\dot{m}_{da} x_1 + \dot{m}_w = \dot{m}_{da} x_2 \quad \Rightarrow \quad \dot{m}_w = \dot{m}_{da} (x_2 - x_1) \quad \ll \text{water} \gg \quad \mathbf{39-26}$$

$$\dot{m}_{da} h_1 + \dot{m}_w h_w + \dot{Q}_s = \dot{m}_{da} h_2 \quad \text{or} \quad \dot{m}_{da} h_1 + \dot{Q}_t = \dot{m}_{da} h_2 \quad \Rightarrow \quad \ll \text{power} \gg \quad \mathbf{39-27}$$

$$\Rightarrow \frac{(h_2 - h_1)}{(x_2 - x_1)} = \frac{\Delta h}{\Delta x} = \frac{\dot{Q}_s}{\dot{m}_w} + h_w = \frac{\dot{Q}_t}{\dot{m}_w} \quad \mathbf{39-28}$$

$$\dot{m}_{da} h_1 + \dot{m}_w h_w + \dot{Q}_s = \dot{m}_{da} h_2 \quad \Rightarrow \quad \dot{m}_w h_w + \dot{Q}_s = \dot{m}_{da} (h_2 - h_1) \quad \Rightarrow \quad \frac{\dot{m}_{da} (h_2 - h_1)}{\dot{m}_w} = \frac{\dot{Q}_s}{\dot{m}_w} + h_w \quad \Rightarrow$$

$$\Rightarrow \frac{\dot{m}_{da} (h_2 - h_1)}{\dot{m}_{da} (x_2 - x_1)} = \frac{\dot{Q}_s}{\dot{m}_w} + h_w \quad \Rightarrow$$

$$\Rightarrow \frac{(h_2 - h_1)}{(x_2 - x_1)} = \frac{\dot{Q}_s}{\dot{m}_w} + h_w = \frac{\dot{Q}_t - \dot{Q}_l}{\dot{m}_w} + h_w = \frac{\dot{Q}_t}{\dot{m}_w} - \frac{\dot{Q}_l}{\dot{m}_w} + h_w = \frac{\dot{Q}_t}{\dot{m}_w} - h_w + h_w = \frac{\dot{Q}_t}{\dot{m}_w}$$

The left side of equation **39-27** represents the total rate of energy addition to the space from all sources. By equations **39-27** and **39-26**, equation **39-28** is obtained [$(h_2 - h_1)/(x_2 - x_1) = \Delta h/\Delta x$] according to which, on psychrometric chart and for a given

state of withdrawn air, all possible states for supply air must lie on a straight line drawn through the state point of withdrawn air, with its direction specified by the numerical value of the *direction coefficient* $\Delta h/\Delta x$. This line represents the process (or condition) line for the given problem. A frequent alternative procedure in establishing the process line is to use the protractor's sensible/total heat ratio scale instead of the $\Delta h/\Delta x$ scale. That is, the *sensible heat ratio* $R = \dot{Q}_s/\dot{Q}_t$, ratio of sensible heat load to total heat load in the space. Being the rate of total heat load \dot{Q}_t the sum of the sensible and latent heat loads ($\dot{Q}_t = \dot{Q}_s + \dot{Q}_l$), the sensible heat ratio can be written:

$$R = \frac{\dot{Q}_s}{\dot{Q}_t} = \frac{\dot{Q}_t - \dot{Q}_l}{\dot{Q}_t} = 1 - \frac{\dot{Q}_l}{\dot{Q}_t} \quad 39-29$$

Being (39-28) $\dot{Q}_t = \dot{m}_w \cdot (\Delta h/\Delta x)$, there is a relationship between the sensible heat ratio R and the direction coefficient $\Delta h/\Delta x$:

$$R = 1 - \frac{\dot{Q}_l}{\dot{Q}_t} = 1 - \frac{\dot{m}_w h_w}{\dot{m}_w \cdot (\Delta h/\Delta x)} = 1 - \frac{h_w}{(\Delta h/\Delta x)} \quad 39-30$$

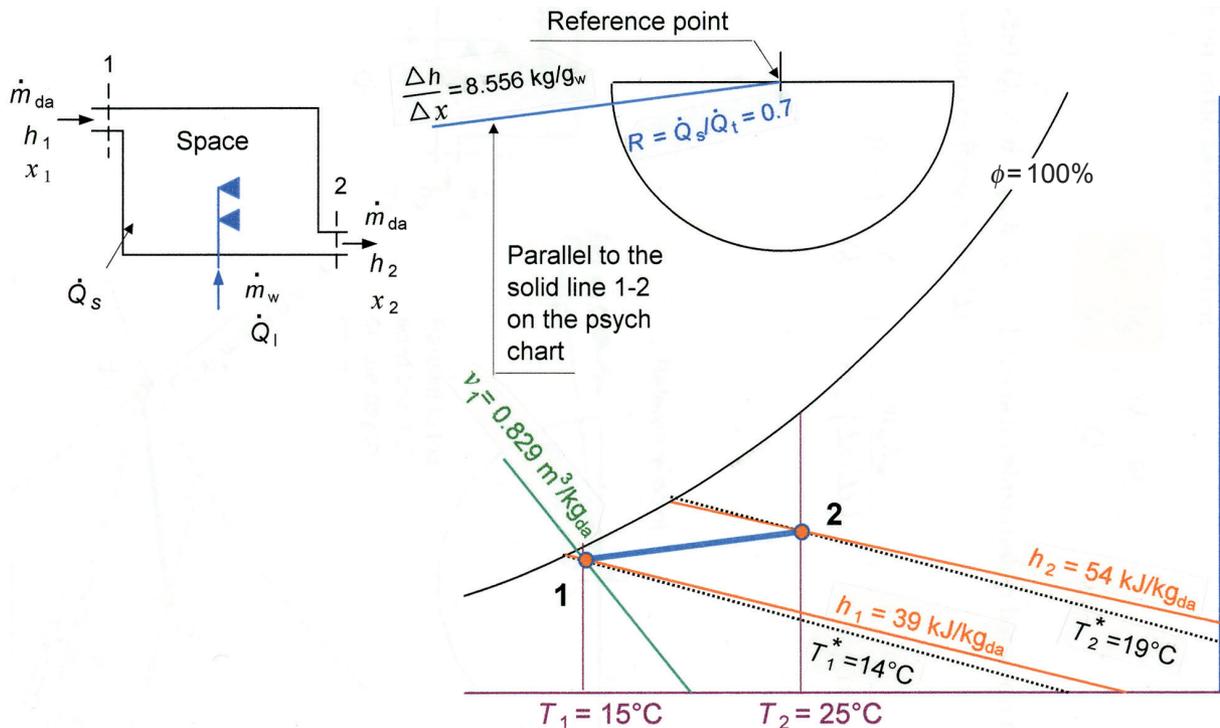


Fig. 39.24 - Scheme of the space to be air conditioned and process on the psychrometric chart; the *process* or *condition* line connecting 1-2 is parallel to the *direction coefficient* $\Delta h/\Delta x = 8.556 \text{ kJ/g}_w$ and gives the process direction.

Example 39.12 Direction coefficient and sensible heat ratio

Moist air is withdrawn from a room at $T_2 = 25^\circ\text{C}$ dry-bulb temperature and $T_2^* = 19^\circ\text{C}$ wet-bulb temperature. The sensible rate of heat gain for the space is $\dot{Q}_s = 9\text{ kW}$. A rate of moisture gain of $\dot{m}_w = 1.5\text{ g}_w/\text{s}$ occurs from the space occupants; this moisture is assumed as saturated water vapor at 30°C . Moist air is introduced into the room at a dry-bulb temperature $T_1 = 15^\circ\text{C}$. Find the direction coefficient $\Delta h/\Delta x$, the sensible heat ratio R , the properties of the states 1 and 2, and the volume flow rate $\dot{V}_{\text{da}1}$ of the supply air.

SOLUTION

Locate state 2 on the psychrometric chart in the intersection of the vertical purple line ($T_2 = 25^\circ\text{C}$) with the diagonal black dotted line ($T_2^* = 19^\circ\text{C}$). The specific enthalpy of the added saturated water vapor at 30°C is (h_g in Table A.3.1) $h_w = 2556.3\text{ kJ/kg}_w = 2.556\text{ kJ/g}_w$; the enthalpy has to be converted in kJ/g_w because the humidity ratio scale on the psychrometric chart is $\text{g}_w/\text{kg}_{\text{da}}$. Determine $\Delta h/\Delta x$ and R ; note that on the protractor $\Delta h/\Delta x = 8.556\text{ kJ/g}_w$ coincides closely with $R = 0.7$. Establish, with the protractor, a reference line of direction $\Delta h/\Delta x$; parallel to this reference line, draw a straight line on the chart through state 2. The intersection of this line with the 15°C dry-bulb temperature line is state 1. Read all properties of states 1 and 2 directly from the chart. The dry mass flow rate \dot{m}_{da} can be calculated with material balance or energy conservation equation. Then, with the specific volume ν_1 , the supply volume $\dot{V}_{\text{da}1}$ is obtained.

$$39-28: \frac{\Delta h}{\Delta x} = \frac{\dot{Q}_s}{\dot{m}_w} + h_w = \frac{9\text{ kW}}{0.0015\text{ kg}_w/\text{s}} + 2556.3 \frac{\text{kJ}}{\text{kg}_w} = 8556.3 \frac{\text{kJ}}{\text{kg}_w} = 8.556 \frac{\text{kJ}}{\text{kg}_w} \quad \blacktriangleleft$$

$$39-24: \dot{Q}_1 = \dot{m}_w h_w = 0.0015 \frac{\text{kg}_w}{\text{s}} \times 2556.3 \frac{\text{kJ}}{\text{kg}_w} = 3.83 \frac{\text{kJ}}{\text{s}}$$

$$39-25: \dot{Q}_t = \dot{Q}_s + \dot{Q}_1 = 9\text{ kW} + 3.83\text{ kW} = 12.83\text{ kW}$$

$$39-29: R = \frac{\dot{Q}_s}{\dot{Q}_t} = \frac{9\text{ kW}}{12.83\text{ kW}} = 0.7 \quad \blacktriangleleft$$

$$\text{State 1: } T_1^* = 14^\circ\text{C} \quad h_1 = 39\text{ kJ/kg}_{\text{da}} \quad x_1 = 9.65\text{ g}_w/\text{kg}_{\text{da}} \quad \nu_1 = 0.829\text{ m}^3/\text{kg}_{\text{da}} \quad \blacktriangleleft$$

$$\text{State 2: } h_2 = 54\text{ kJ/kg}_{\text{da}} \quad x_2 = 11.40\text{ g}_w/\text{kg}_{\text{da}} \quad \blacktriangleleft$$

$$39-26: \dot{m}_w = \dot{m}_{\text{da}}(x_2 - x_1) \Rightarrow \dot{m}_{\text{da}} = \frac{\dot{m}_w}{x_2 - x_1} = \frac{0.0015\text{ kg}_w/\text{s}}{(0.0114 - 0.00965)\text{ kg}_w/\text{kg}_{\text{da}}} = 0.857 \frac{\text{kg}_{\text{da}}}{\text{s}}$$

$$39-27: \dot{m}_{\text{da}}(h_2 - h_1) = \dot{Q}_1 + \dot{Q}_s \Rightarrow \dot{m}_{\text{da}} = \frac{\dot{Q}_1 + \dot{Q}_s}{h_2 - h_1} = \frac{3.83\text{ kW} + 9\text{ kW}}{(54 - 39)\text{ kJ/kg}_{\text{da}}} = 0.855 \frac{\text{kg}_{\text{da}}}{\text{s}}$$

$$\dot{V}_{\text{da}1} = \dot{m}_{\text{da}} \nu_1 = 0.855\text{ kg}_{\text{da}}/\text{s} \times 0.829\text{ m}^3/\text{kg}_{\text{da}} = 0.709\text{ m}^3/\text{s} \quad \blacktriangleleft$$

39.12.7 Air conditioning

The human comfort depends mainly on three factors meeting proper intervals: dry-bulb temperature $T = 20 \div 26$ °C, relative humidity $\phi = 40 \div 60\%$ and ventilation $v = 0.05 \div 0.3$ m/s. The air conditioning must guarantee that these factors – temperature, relative humidity, air velocity – meet the above intervals in civil and industrial buildings. For a space, the basic scheme of the air conditioning system is as follows (Figure 39.25):

- the space A – air withdrawn from this space, where assigned ambient conditions have to be maintained, is partly discharged and partly recirculated;
- the mixer M – the flow of recirculated air \dot{m}_R is mixed with the outside air \dot{m}_E in a closely adiabatic way;
- the air conditioner C – here the air is subjected to specific operations as, for instance, refrigeration and dehumidification;
- the fan V – it allows air flowing in the circuit; because of the small power absorbed, its contribution to the energy balance is usually neglected (Paragraph 39.12.1).

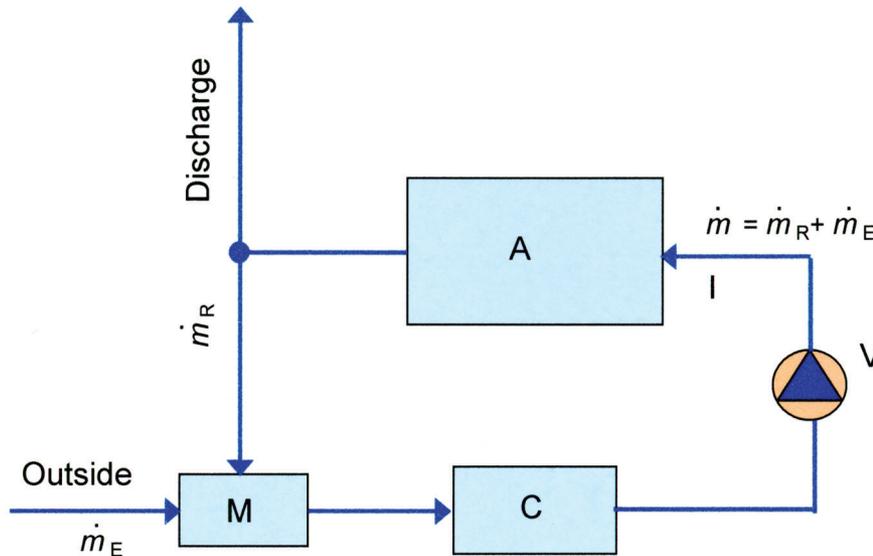


Fig. 39.25 - Scheme of air conditioning plant.

Once fixed temperature and relative humidity to be established in room A, the design of the air conditioning system is accomplished in two steps:

1. at first, determine the heat loads, i.e. the rate of heat transfer between the room A and outside air E; then, calculate those quantities (mass flow rate, temperature and relative humidity) that the intake air (state I) must meet to comply with the project requirements;
2. define processes to which the intake air must be subjected to meet the state I conditions.

Step 2 has to be followed by another step with sizing of the air piping network and of the selection of instrumentation and control systems.

The sensible heat load \dot{Q}_s exchanged between the space A, to be conditioned, and outside air E is the sum of four terms:

1. $\dot{Q}_{s,AE}$ (>0 in summer and <0 in winter) – sensible load exchanged between space A and outside E because of temperature difference and radiation through all walls, doors and roof; this heat power $\dot{Q}_{s,AE} = \sum U_i A_i (T_{E,i} - T_A)$ has to be intended (27-19) as summation of a number i of overall heat transfer coefficients U for area A and related outside temperature T_E ;
2. $\dot{Q}_{s,pers}$ (always >0) – sensible load due to occupants; it is obtained by multiplying the N persons for the values shown in Table 39.3;
3. $\dot{Q}_{s,sources}$ (always >0) – sensible load produced by inside sources of power as lighting, machinery, etc.;
4. $\dot{Q}_{s,inf}$ – sensible load linked to dry air mass flow rate $\dot{m}_{da,inf}$ entering the space because of outside air infiltration, $\dot{Q}_{s,inf} = \dot{m}_{da,inf} c_{p,da} (T_E - T_A)$.

We saw (39-24) that the latent heat load is $\dot{Q}_l = \dot{m}_w h_w$ with \dot{m}_w as vapor mass flow rate entering the space and h_w as specific enthalpy of saturated vapor at room temperature (≈ 2550 kJ/kg_w). Unless other sources linked to vapor generation within the space and to outside air infiltration, \dot{Q}_l is obtained by multiplying the latent heat of each occupant (Table 39.3) for the N persons.

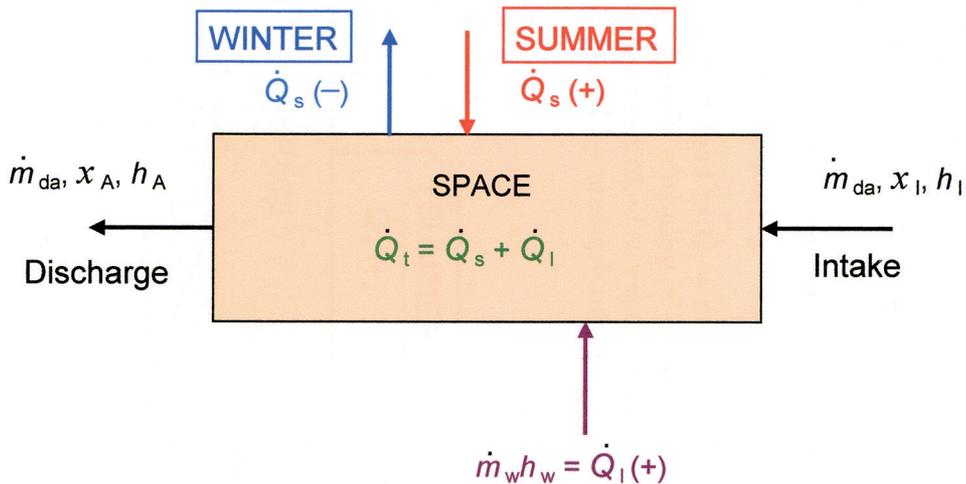


Fig. 39.26 - Energy exchange between space A to be conditioned and surroundings E.

Once sensible heat \dot{Q}_s and latent heat loads \dot{Q}_l are known, the total heat load is obtained $\dot{Q}_t = \dot{Q}_s + \dot{Q}_l$; then, with sensible heat ratio R (39-29), it is possible to determine the process line on psychrometric chart (Paragraph 39.12.7).

The calculation scheme for step 1 is:

- a) being outside air conditions T_E and ϕ_E known, the room conditions are fixed by taking into account that the human comfort conditions (even more imposed by law) require

Table 39.3

Heat loads per person (sensible \dot{Q}_s and latent \dot{Q}_l)

These average figures can change passing from summer to winter; for instance, the “Moderately active office work” with summer figures of $\dot{Q}_s = 75$ W/pers and $\dot{Q}_l = 55$ W/pers, should be taken as $\dot{Q}_s = 70$ W/pers and $\dot{Q}_l = 50$ W/pers in winter.

Degree of activity		\dot{Q}_s [W/pers]	\dot{Q}_l [W/pers]
Seated at theater	Theater, matinée	65	30
Seated at theater, night	Theater, night	70	35
Seated, very light work	Offices, hotels, apartments	70	45
Moderately active office work	Offices, hotels, apartments	75	55
Standing, light work, walking	Department store, retail store	75	55
Walking, standing	Drug store, bank	75	70
Sedentary work	Restaurant	80	80
Light bench work	Factory	80	140
Moderate dancing	Dance hall	90	160
Walking 4,8 km/h, light machine work	Factory	110	185
Heavy work	Factory	170	255
Heavy machine work, lifting	Factory	185	285
Athletics	Gymnasium	210	315

$T_A = 26$ °C - $\phi_A = 50\%$ in summer, and $T_A = 20$ °C - $\phi_A = 50\%$ in winter. Based on the design values of dry-bulb temperature and relative humidity, other properties of states A and E are calculated or read on psychrometric chart;

- sensible \dot{Q}_s and latent \dot{Q}_l heat load, and total heat load \dot{Q}_t are calculated. Then the sensible heat ratio ($R = \dot{Q}_s/\dot{Q}_t$), that will be used to draw the process line, is obtained;
- it is known, because fixed or calculated from other assigned data, a property of intake state I. It could be the temperature T_I , having a value able to satisfy the inequalities $(T_A - T_I) \leq 8 \div 12$ °C in summer and $(T_I - T_A) \leq 12 \div 20$ °C in winter. The point I has to be neither too much close to point A, because the air mass flow \dot{m}_{da} should result too much high, nor too much distant from A, because of the thermal shock of a mass flow rate entering a zone with a temperature T_I so different from T_A . Another property could be the humidity ratio x_I that, if not fixed initially, is calculated (39-26) with $x_I = x_A - \dot{m}_w/\dot{m}_{da}$, as a function of mass flow rate of vapor $\dot{m}_w = \dot{Q}_l/h_w$ (39-24) and dry air \dot{m}_{da} (39-28);
- the dry air mass flow \dot{m}_{da} can be approximately calculated as a function of sensible heat load \dot{Q}_s and air specific heat at constant pressure $c_{p,da} = 1.006$ kJ/(kg·°C). The formula also should be used to obtain the intake temperature T_I ;

$$\dot{Q}_s \cong \dot{m}_{da} c_{p,da} (T_A - T_I) \Rightarrow \dot{m}_{da} \cong \frac{\dot{Q}_s}{c_{p,da} (T_A - T_I)} \quad \text{or} \quad T_I \cong T_A - \frac{\dot{Q}_s}{\dot{m}_{da} c_{p,da}} \quad \mathbf{39-31}$$

- e) enthalpy of state I can be calculated with **39-28** [$h_I = h_A + (\dot{Q}_t / \dot{m}_w)(x_I - x_A)$] or read on psychrometric chart by establishing the point I at the intersection of the process line drawn from A with the other property of state I defined in (c).

In the step 2, the type of the air conditioning system (see, for instance, the scheme of *Figure 39.27*) is defined:

- a) the air mass flow \dot{m}_{daE} taken from the outside depending on the number of the occupants. The flow is $\dot{m}_{daE} = N \cdot \dot{m}_{daE/pers}$ with N persons and $\dot{m}_{daE/pers}$ outside air change per person; $\dot{m}_{daE/pers}$ is equal to $5 \div 16 \text{ g}_{da}/(\text{s} \cdot \text{pers}) = 18 \div 60 \text{ kg}_{da}/(\text{h} \cdot \text{pers})$, where the highest values refer to bar and hotel rooms. Thus, the mass flow rate of recirculated air is the total mass flow of dry air \dot{m}_{da} minus the air change: $\dot{m}_{daR} = \dot{m}_{da} - \dot{m}_{daE}$. The air recirculation can not be actuated in buildings where is a risk of infection (hospitals) or explosion (chemical factories). Where possible, the partial air recirculation is strongly suggested because of the large energy saving (*Figure 39.27*);
- b) in step 1, the properties of states A (space to be conditioned) and E (outside air) were defined. The mass flow rate \dot{m}_{daR} of the recirculated air mixes in M with the outside air \dot{m}_{daE} ; note that the properties of recirculated air are the same as those of state A because coming from space A. By writing (**39-22**) the steady-flow energy and water balance equations of the moist air leaving the mixer (state M), humidity ratio x_M and enthalpy h_M are found as average weight on dry air mass flow rate \dot{m}_{da} . Once x_M and h_M values are known, the temperature T_M can be calculated with **39-15'** or the approximate formula:

$$T_M \cong \frac{\dot{m}_{daE} T_E + \dot{m}_{daA} T_A}{\dot{m}_{daE} + \dot{m}_{daA}} = \frac{\dot{m}_{daE} T_E + \dot{m}_{daA} T_A}{\dot{m}_{da}} \quad (39-15'')$$

Or, by using the psychrometric chart, locate point M on the line E-A, because the ratio of segments EM and EA is equal to mass flow rates (see caption of *Figure 39.21*):

$$\frac{EM}{EA} = \frac{\dot{m}_{daR}}{\dot{m}_{da}} \Rightarrow EM = \frac{\dot{m}_{daR}}{\dot{m}_{da}} EA$$

- c) In general the air conditioning process is represented by cooling with dehumidification + heating in summer and heating + humidification in winter. The required power is obtained by multiplying the dry air flow \dot{m}_{da} for the enthalpy change. Winter processes are (*Figure 39.28*): sensible pre-heating from state M to state R at constant humidity ratio ($x_R = x_M$), humidification at constant enthalpy ($h_R = h_S$) until it intersects the saturation curve (state S) and sensible post-heating to state I. However, only $x_R = x_M$ is known and this information is not sufficient to locate R. It is then necessary to go backwards from the already determined point I. The post-heating S-I is a process with constant humidity ratio ($x_S = x_I$) of a mixture that would leave the humidifier as saturated condition ($\phi = 100\%$); this would be the straight way to locate the point S on psychrometric chart. In practice, a complete adiabatic saturation is not feasible, because the cooler has a saturation effectiveness less than 100% ($\phi = 70 \div 90\%$). For instance, assuming that the adiabatic saturation process is stopped at the relative humidity $\phi = 70\%$, the enthalpy of the state S is read as intersection of the humidity ratio $x_S = x_I$ with $\phi = 70\%$ curve.

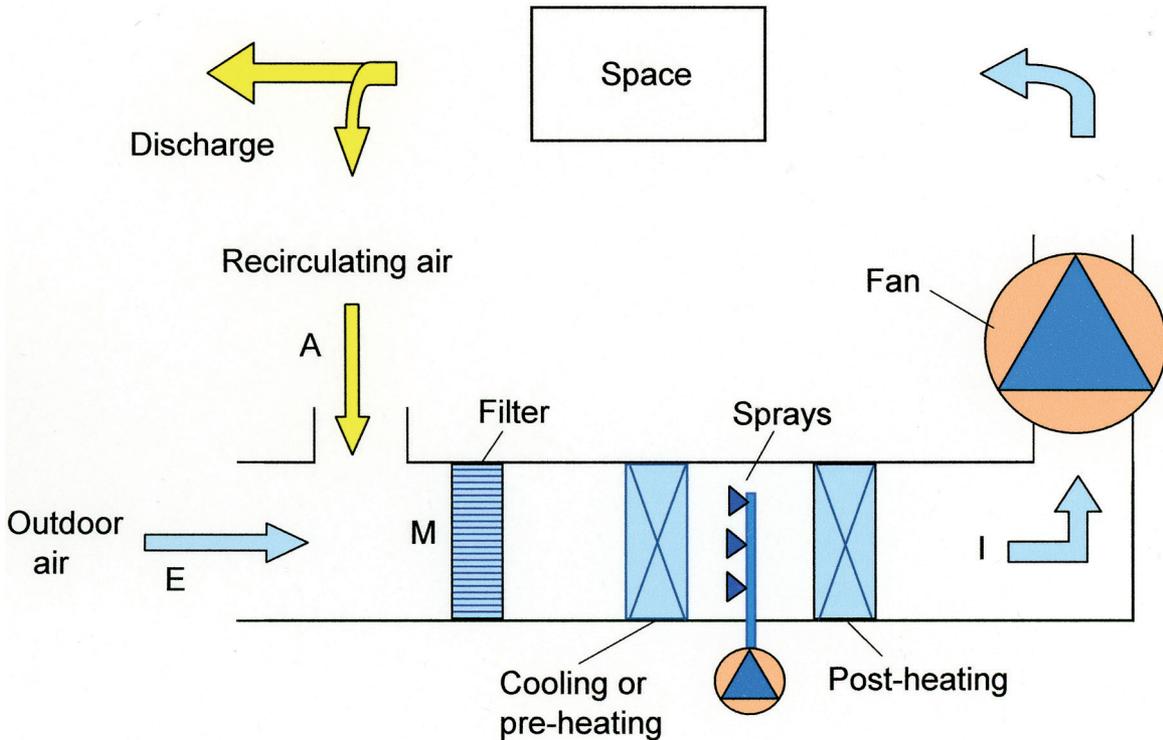


Fig. 39.27 - Scheme of an all-air ducted single-zone fan system where outdoor air mixes with recirculating air before entering the air conditioner.

39.13 Cooling towers

39.13.1 Description

In a vapor cycle, heat transfer from the working fluid takes place during flow through the condenser. In units of large capacity, this transfer typically is from the working fluid to circulating water passing through the condenser. Flow rates for condenser circulating water are very large. Thus a power station of the given capacity would be located near a large body of water or a river and once-through cooling would be used. Such cooling involves taking water from the river, passing it through the condenser tubes, and discharging the heated water back into the river with a temperature increase up to 10 °C, thus altering in a significant way the ecologic equilibrium of the river and between the river and surrounding region.

Large canals where the water is cooled by evaporation and convection are one alternative to once-through cooling. One more efficient method to cool circulating water is the use of cooling towers. There are *dry cooling towers*, that are closed water-to-air surface heat exchangers being no contact between working fluid and air, and *evaporative mechanical-* and *natural-draft cooling towers*, where hot water is in close contact with the outside air.

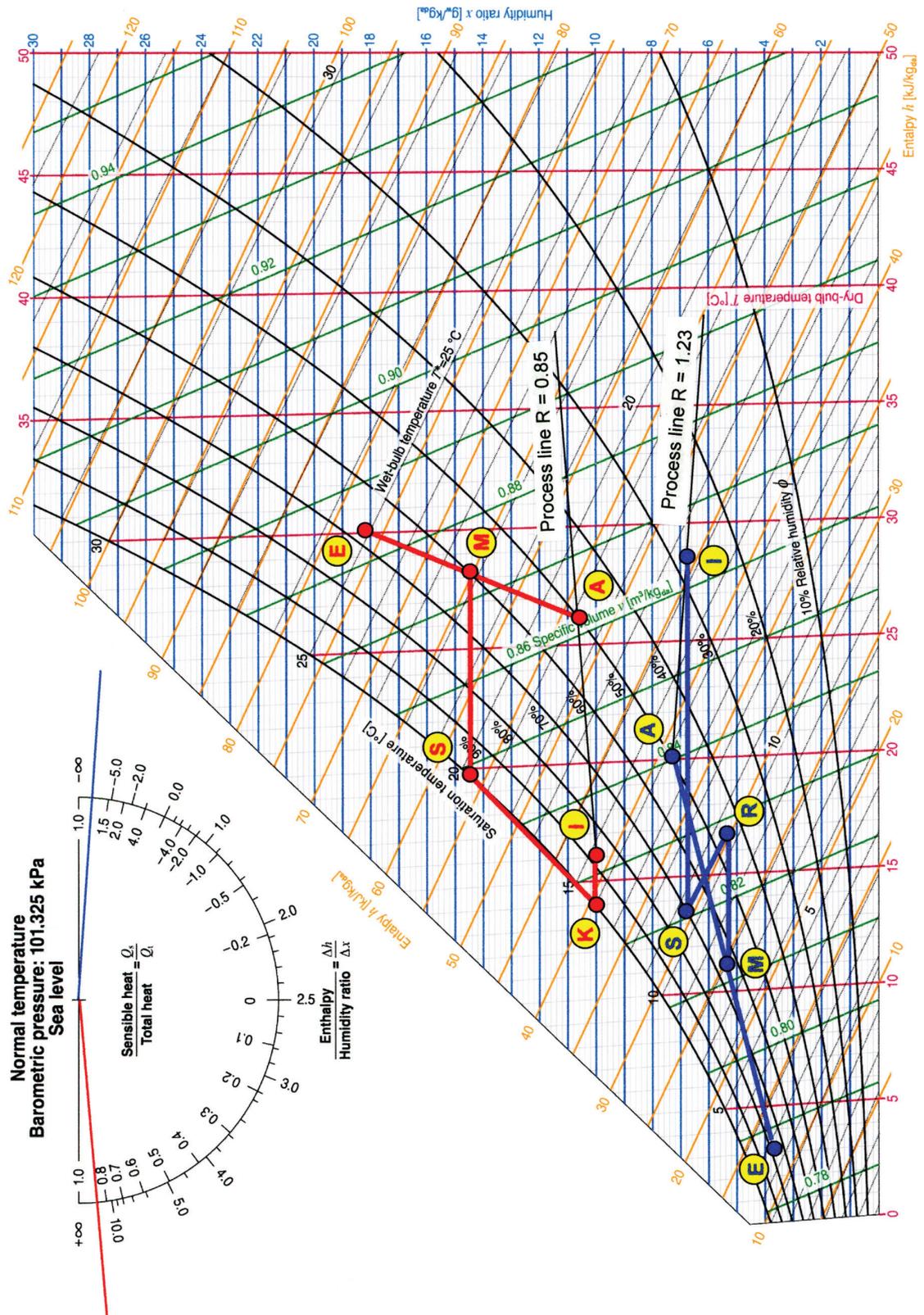


Fig. 39.28 - Air conditioning process in summer and in winter (*Tema 2014T*).

In a cooling tower (*Figure 39.29*) ambient air enters at A. Warm moist air ($\phi \approx 100\%$) is discharged at B. Warm water from the condenser of the power plant enters at the top as a mist at C, and the cooled water at D is mixed with make-up water and returned to the condenser. The makeup water flow rate equals the rate at which water evaporates in the tower.

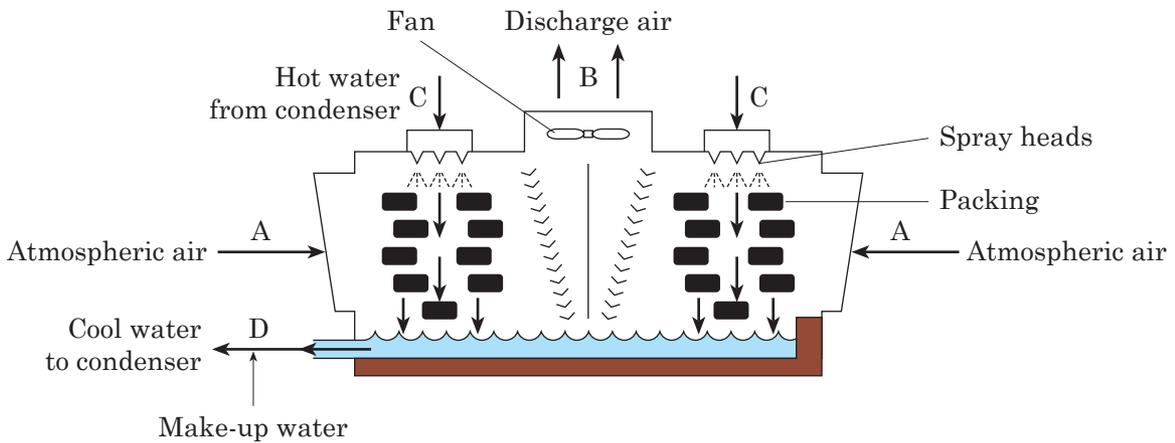


Fig. 39.29 - Induced draft, double-flow crossflow tower.

Figure 39.30 shows a scheme of a natural-draft evaporative cooling tower. Natural draft towers depend on buoyant effect to move the air through the tower. While fans are not used, air warmed by the hot water in the lower part of the tower rises to create a natural draft through the tower. In general, the hot water falls down over fill surfaces, which help increase the contact time between the water and the air thus maximize the contact between the two. Due to large size of these towers, only used by utility power stations, they are used for flow rates above $12 \text{ m}^3/\text{s}$.

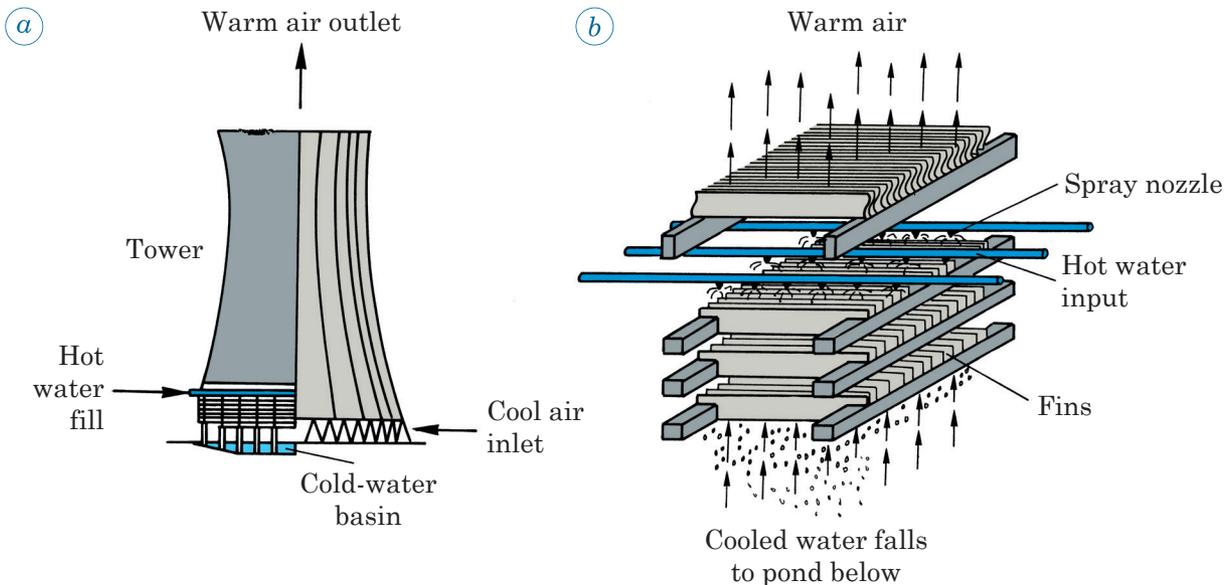


Fig. 39.30 - A natural draft evaporative cooling tower:
a) external view; b) inside tower.

An inherent disadvantage of evaporative towers, both mechanical and natural draft, is that the discharge air is saturated and the temperature of the surrounding atmosphere may be well below the dew-point temperature of the discharge air, causing condensed vapor deposition on lands and building close to the towers.

39.13.2 Conservation equations

The analysis of the thermodynamic process in the evaporative tower (*Figure 39.31*) is conducted by assuming that there are no heat and work exchange between the tower walls and the outside air and that kinetic energy change can be neglected.

$$\left. \begin{aligned} \dot{m}_{\text{da}}x_1 + \dot{m}_{\text{w3}} &= \dot{m}_{\text{da}}x_2 + \dot{m}_{\text{w4}} \Rightarrow \dot{m}_{\text{w4}} = \dot{m}_{\text{w3}} - \dot{m}_{\text{da}}(x_2 - x_1) &<< \text{water} >> \\ \dot{m}_{\text{da}}h_1 + \dot{m}_{\text{w3}}h_{\text{w3}} &= \dot{m}_{\text{da}}h_2 + \dot{m}_{\text{w4}}h_{\text{w4}} \Rightarrow \dot{m}_{\text{da}} = \frac{\dot{m}_{\text{w3}}(h_{\text{w4}} - h_{\text{w3}})}{(h_1 - h_2) + (x_2 - x_1)h_{\text{w4}}} &<< \text{power} >> \end{aligned} \right\} \mathbf{39-32}$$

$$\begin{aligned} \dot{m}_{\text{da}}h_1 + \dot{m}_{\text{w3}}h_{\text{w3}} &= \dot{m}_{\text{da}}h_2 + \dot{m}_{\text{w4}}h_{\text{w4}} \Rightarrow \dot{m}_{\text{da}}(h_1 - h_2) + \dot{m}_{\text{w3}}h_{\text{w3}} - \dot{m}_{\text{w4}}h_{\text{w4}} = 0 \Rightarrow \\ \Rightarrow \dot{m}_{\text{da}}(h_1 - h_2) + \dot{m}_{\text{w3}}h_{\text{w3}} - [\dot{m}_{\text{w3}} - \dot{m}_{\text{da}}(x_2 - x_1)]h_{\text{w4}} &= 0 \Rightarrow \\ \Rightarrow \dot{m}_{\text{da}}(h_1 - h_2) + \dot{m}_{\text{da}}(x_2 - x_1)h_{\text{w4}} &= \dot{m}_{\text{w3}}(h_{\text{w4}} - h_{\text{w3}}) \Rightarrow \dot{m}_{\text{da}} = \frac{\dot{m}_{\text{w3}}(h_{\text{w4}} - h_{\text{w3}})}{(h_1 - h_2) + (x_2 - x_1)h_{\text{w4}}} \end{aligned}$$

The make-up water $\Delta\dot{m}_{\text{w}}$ (i.e. the water that must be added to the circulating water system in order to compensate for water losses such as evaporation, drift loss, blow-out, blow-down, etc.) increases the humidity ratio in a way that the humidity ratio x_2 of the air leaving the tower is higher than x_1 of the air entering the tower. Based on the material balance equation (39-32), $\Delta\dot{m}_{\text{w}}$ due to the evaporation within the air is:

$$\Delta\dot{m}_{\text{w}} = \dot{m}_{\text{w3}} - \dot{m}_{\text{w4}} = \dot{m}_{\text{da}}(x_2 - x_1) \quad \mathbf{39-33}$$

The minimum temperature T_4 to be reached by water leaving the evaporative tower is the wet-bulb temperature of the intake moist air T_1^* ; this is a rather theoretical case corresponding to an adiabatic saturation process (see comment in the *Example 39.13*). Then the *effectiveness* ε of the evaporative tower can be defined as the ratio between the heat power subtracted to water \dot{Q} and the maximum theoretical power \dot{Q}_{max} :

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} \cong \frac{T_3 - T_4}{T_3 - T_1^*} \quad \mathbf{39-34}$$

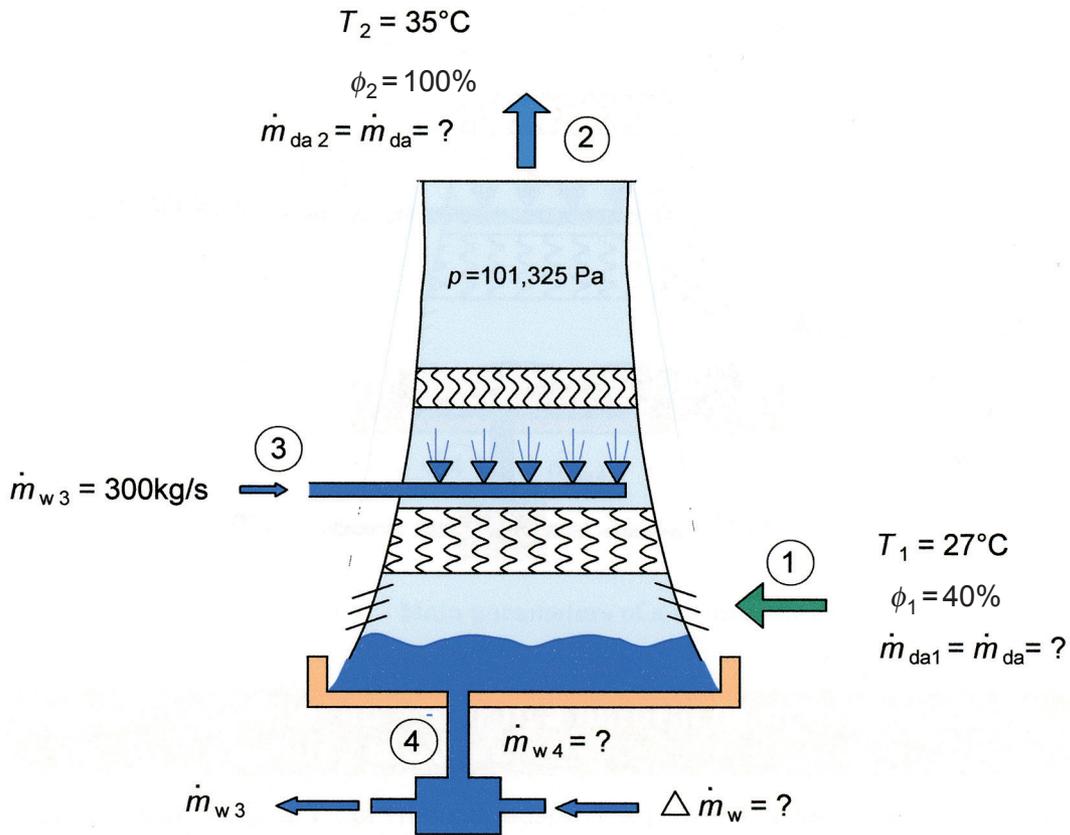


Fig. 39.31 - Main parameters of a cooling tower.

Example 39.13 Make-up water in a natural draft evaporative cooling tower

The cooling water, coming from the condenser of a power station, is sent to a number of natural-draft cooling towers, each of them taking care of a water mass flow $\dot{m}_{w3} = 300 \text{ kg/s}$ (Figure 39.31). Water leaves the condenser at temperature $T_3 = 40^\circ\text{C}$ and is cooled down to the temperature $T_4 = 30^\circ\text{C}$ by means of atmospheric air entering the tower with $T_1 = 27^\circ\text{C}$ - $\phi_1 = 40\%$ and leaves the tower with $T_2 = 35^\circ\text{C}$ - $\phi_2 = 100\%$. Find the make-up to total water flow ratio $\Delta\dot{m}_w/\dot{m}_w$ and the heat power \dot{Q} .

SOLUTION

Being assigned dry-bulb temperature T and relative humidity ϕ of the moist air entering and leaving the tower, other properties are obtained by calculation or from the psychrometric chart:

$$\mathbf{39-13:} \quad p_{ws1} \cong 1000 \cdot e^{\left(16.6536 - \frac{4030.183}{T + 235}\right)} = 1000 \cdot e^{\left(16.6536 - \frac{4030.183}{27^\circ\text{C} + 235}\right)} = 3365 \text{ Pa}$$

$$p_{ws2} \cong 1000 \cdot e^{\left(16.6536 - \frac{4030.183}{T + 235}\right)} = 1000 \cdot e^{\left(16.6536 - \frac{4030.183}{35^\circ\text{C} + 235}\right)} = 5624 \text{ Pa}$$

$$39-14: x_1 = 0.622 \frac{\phi_1 p_{ws1}}{p - \phi_1 p_{ws1}} = 0.622 \frac{0.4 \times 3365 \text{ Pa}}{101,325 \text{ Pa} - 0.4 \times 3365 \text{ Pa}} = 0.00837 \frac{\text{kg}_w}{\text{kg}_{da}}$$

$$x_2 = 0.622 \frac{\phi_2 p_{ws2}}{p - \phi_2 p_{ws2}} = 0.622 \frac{1.0 \times 5624 \text{ Pa}}{101,325 \text{ Pa} - 1.0 \times 5624 \text{ Pa}} = 0.03655 \frac{\text{kg}_w}{\text{kg}_{da}}$$

$$39-15: h \cong 1.006 \cdot T + x (2501 + 1.86 \cdot T)$$

$$h_1 \cong 1.006 \times 27 \text{ °C} + 0.00837 \frac{\text{kg}_w}{\text{kg}_{da}} \times (2501 + 1.86 \times 27 \text{ °C}) = 48.52 \frac{\text{kJ}}{\text{kg}_{da}}$$

$$h_2 \cong 1.006 \times 35 \text{ °C} + 0.03655 \frac{\text{kg}_w}{\text{kg}_{da}} \times (2501 + 1.86 \times 35 \text{ °C}) = 129.00 \frac{\text{kJ}}{\text{kg}_{da}}$$

$$h_{w3} = c_w \cdot T_3 = 4.186 \frac{\text{kJ}}{\text{kg}_w \cdot \text{°C}} \times 40 \text{ °C} = 167.4 \frac{\text{kJ}}{\text{kg}_w} \quad (h_f = 167.57 \text{ kJ/kg}_w, \text{ Table A.3.1})$$

$$h_{w4} = c_w \cdot T_4 = 4.186 \frac{\text{kJ}}{\text{kg}_w \cdot \text{°C}} \times 30 \text{ °C} = 125.6 \frac{\text{kJ}}{\text{kg}_w} \quad (h_f = 125.79 \text{ kJ/kg}_w, \text{ Table A.3.1})$$

Moist air						Water	
State	T [°C]	φ [-]	p _{ws} [Pa]	x [kg _w /kg _{da}]	h [kJ/kg _{da}]	T [°C]	h _w [kJ/kg _w]
1	27	0.4	3361	0.00837	48.52		
2	35	1.0	4243	0.03655	129.00		
3						40	167.4
4						30	125.6

Black values are known data; red values are calculated.

$$39-32: \dot{m}_{da} = \frac{\dot{m}_{w3} (h_{w4} - h_{w3})}{(h_1 - h_2) + (x_2 - x_1) h_{w4}} =$$

$$= \frac{300 \frac{\text{kg}_w}{\text{s}} \times \left(125.6 \frac{\text{kJ}}{\text{kg}_w} - 167.4 \frac{\text{kJ}}{\text{kg}_w} \right)}{(48.52 - 129) \frac{\text{kJ}}{\text{kg}_{da}} + (0.03655 - 0.00837) \frac{\text{kg}_w}{\text{kg}_{da}} \times 125.6 \frac{\text{kJ}}{\text{kg}_w}} = 163.0 \frac{\text{kg}_{da}}{\text{s}}$$

$$39-33: \Delta \dot{m}_w = \dot{m}_{w3} - \dot{m}_{w4} = \dot{m}_{da} (x_2 - x_1) = 175.3 \text{ kg}_{da}/\text{s} \times (0.03655 - 0.00855) \text{ kg}_w/\text{kg}_{da} = 4.94 \text{ kg}_w/\text{s}$$

$$\frac{\Delta \dot{m}_w}{\dot{m}_w} = \frac{4.94 \text{ kg}_w/\text{s}}{300 \text{ kg}_w/\text{s}} = 0.0164 \approx 1.6\% \quad \blacktriangleleft$$

The heat rate \dot{Q} that the tower is able to subtract from the air is the product of water mass flow $\dot{m}_w = \dot{m}_{w3}$ for the difference between enthalpy entering and leaving the tower $h_{w3} - h_{w4}$.

$$\dot{Q} = \dot{m}_w (h_{w3} - h_{w4}) = 300 \text{ kg}_w/\text{s} \times (167.4 - 125.6) \text{ kJ/kg}_w = 12,540 \text{ kW} \quad \blacktriangleleft$$

COMMENT In this Example the temperature of water leaving the tower ($T_4 = 30^\circ\text{C}$) is higher than the temperature of air entering the tower ($T_1 = 27^\circ\text{C}$). However T_4 could be lower than T_1 because, due to the evaporation mechanism, the water can be cooled below the entering-air dry-bulb temperature. In fact, the water could in theory reach the entering-air wet-bulb temperature. The minimum temperature of the water leaving the tower is $T_1^* = 17.8^\circ\text{C}$, read on the psychrometric chart at $T_1 = 27^\circ\text{C}$ and $\phi_1 = 40\%$. The theoretical value can be only achieved with an evaporative cooling tower with no heat exchange with outside (full adiabatic process) and with an infinite water-to-air exchange surface (infinite efficiency of heat transfer). In practice, the minimum temperature that can be achieved by water leaving the tower is higher of about 6°C than the wet-bulb temperature of the entering air. Here, the minimum water temperature would be $17.8^\circ\text{C} + 6^\circ\text{C} = 23.8^\circ\text{C}$ against $T_4 = 30^\circ\text{C}$, as assigned by the Example. This is accounted by the evaporative tower effectiveness:

$$\mathbf{39-34:} \quad \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} \cong \frac{T_3 - T_4}{T_3 - T_1^*} = \frac{40^\circ\text{C} - 30^\circ\text{C}}{40^\circ\text{C} - 17.8^\circ\text{C}} = 0.45$$